Nanofluidics and electrokinetics

Anne-Laure Biance

Content

Content

1) Introduction

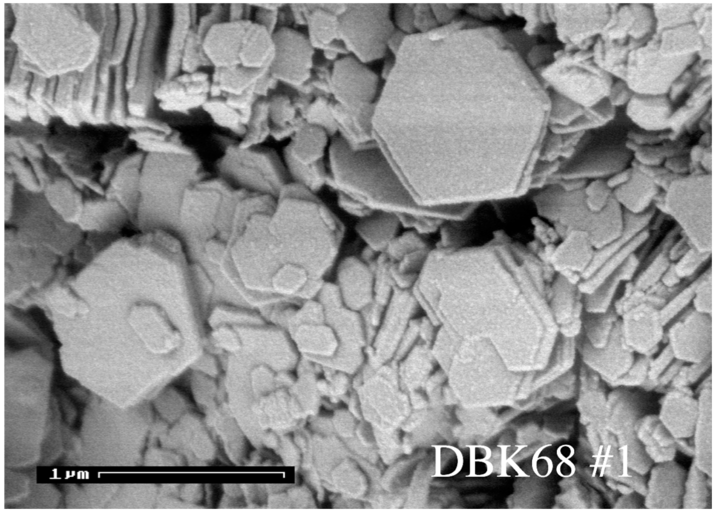
Nanofluidics:

fluid flows in channels with a least one nanometric dimension

(Here: fluid = water / nanometric dimension = 1 to 500 nm)

1) Introduction

From soils…



SEM picture of the soil of a kaolin mine in middle Georgia. http://clay.uga.edu/courses/8550/DBK.html

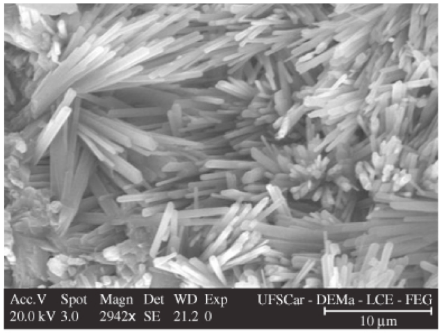
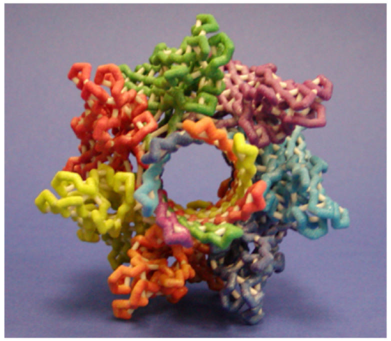


Figure 9. SEM and secondary electron images. Matrix without fibers.

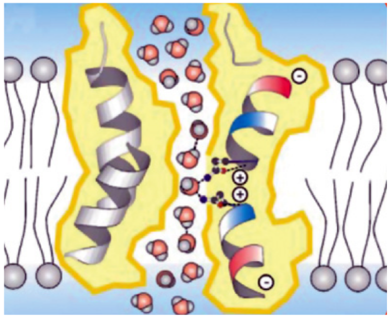
Gypsum

1) Introduction

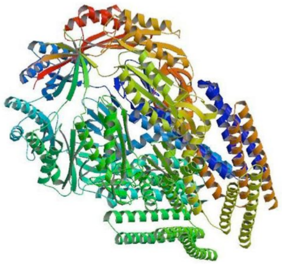
… to living cells.



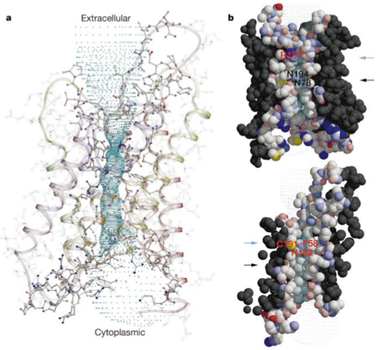
α-hémolysine



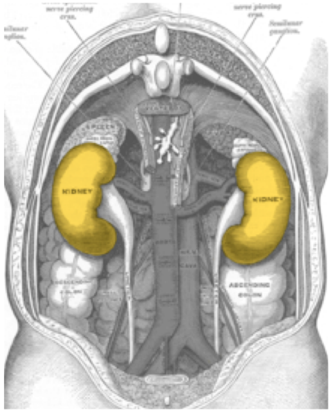
aquaporines



Mechanosensitive channels…



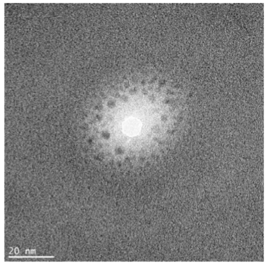
Sui et al., Nature 2001



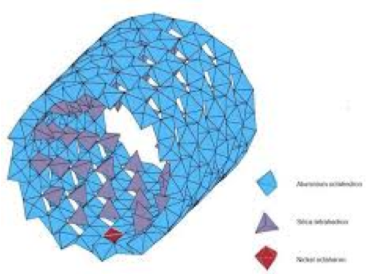
1) Introduction

What is new?

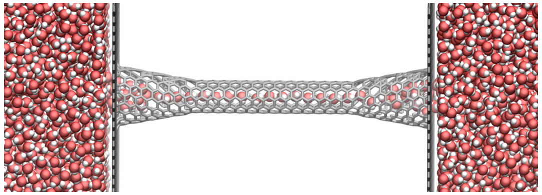
- New synthetic systems: zeolites, imogolytes, nanotubes, nanofabrication



LPN, Marcoussis



- New tools for study: optical and electronic microscopes, electrical detection, surface force apparatus, MD simulations.



Gravelle S et al.*The Journal of Chemical Physics* 2014; **141**: 18C526.

- New applications

Patch-clamp
Neher and Sakmann 1976

1) Introduction

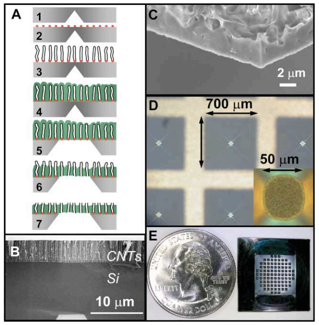
New applications

- Bio-analysis with single molecule resolution
- Specific hydrodynamic transport (super-lubricity)
- Specific ionic transport
- Applications in energy conversion

1) Introduction

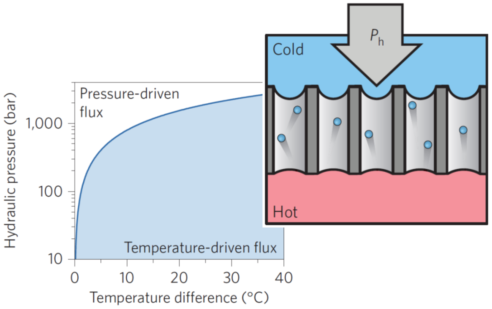
New applications

Filtering: Giant permeability CNT membranes



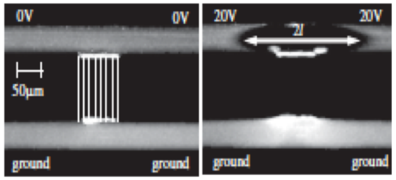
Holt et al Science 2006

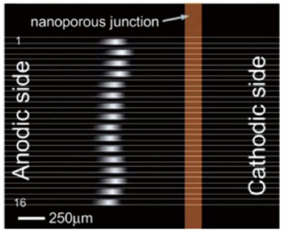
Energy conversion



Straub et al., Nature Energy 2016

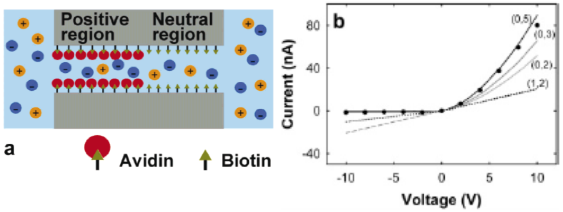
Diagnosis: Concentration polarization





Kim et al., PRL 2007
Son et al., BioChip J. 2016

Non linear effects: nanofluidic diode

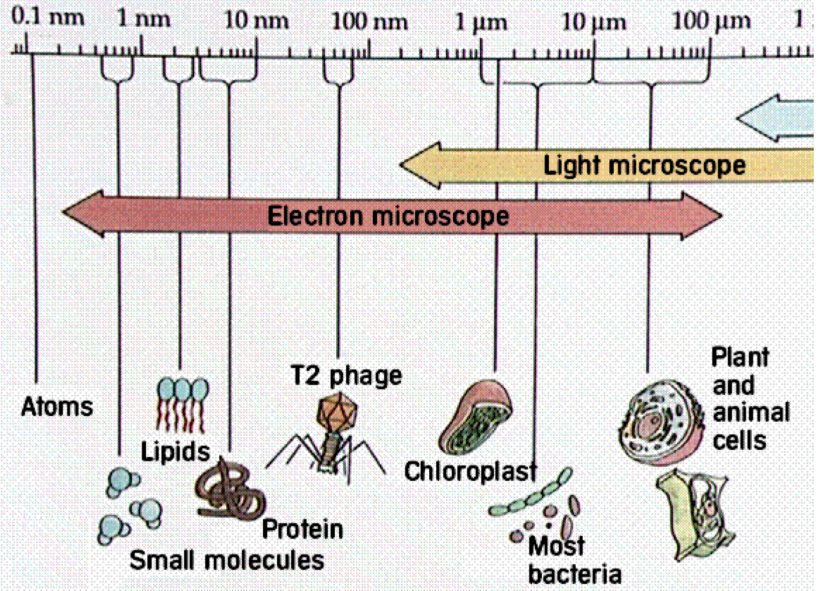


Karnik et al., NanoLett. 2007

1) Introduction

Limits of the macroscopic hydrodynamic description

#limit 1: Size of object / size of the pore

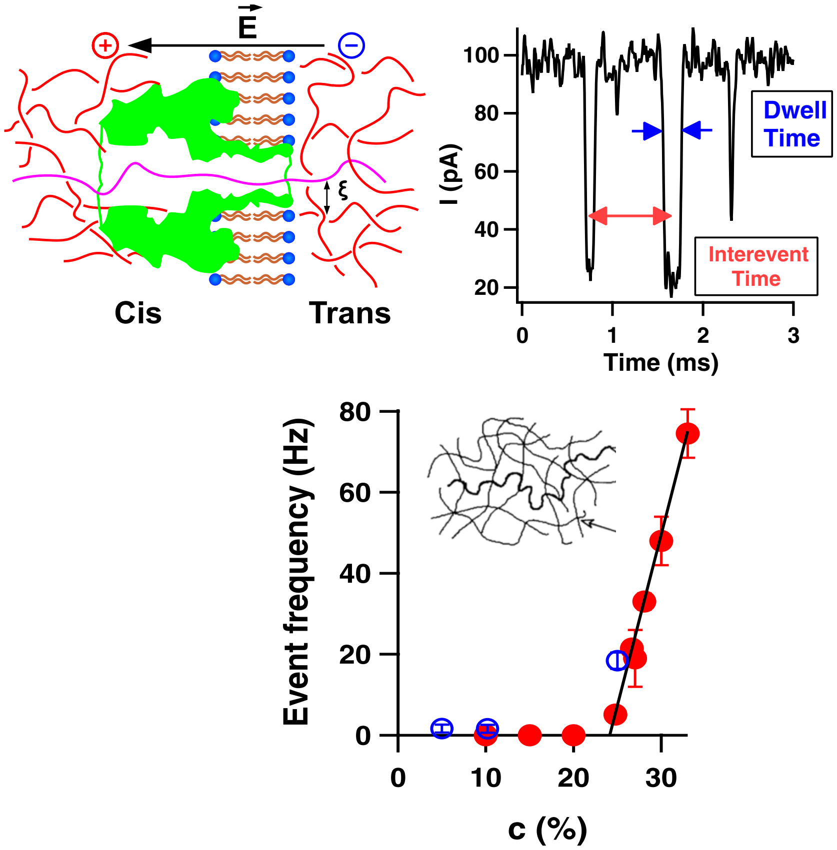


#limit 2: fluctuation and thermal relaxation timescale

- Observations (experiments and simulations):
  - Limits of viscosity definition: 1 nm for water...
  - Other transport coefficients (diffusion): larger…

1) Introduction

#1 Polymers: entropical cost of confinement

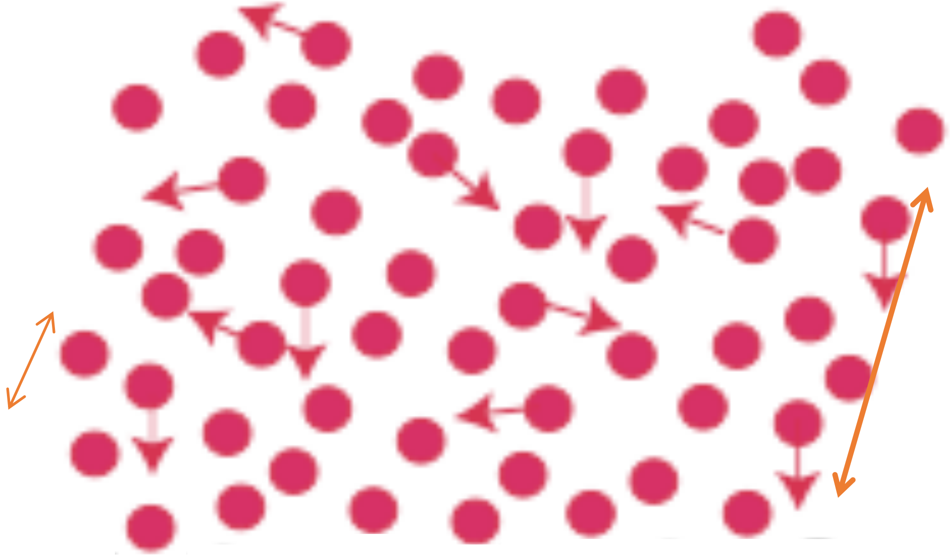


Oukhaled et al., PRL 2012

1) Introduction

#2 Limits of the continuum description: fluctuations

N molecules : fluctuations expected of the order of $\frac{1}{\sqrt{N}}$



Reasonable experimental threshold 10% (100 molecules):

$$L \sim 1nm$$

Water

1) Introduction

QUIZZ

Is continuum hydrodynamics valid for water in a:

a) 1 µm channel?

b) 1 nm channel?

c) 0.1 nm channel?

Content

1) Introduction

2) Mass transport

3) Electrokinetics

4) Concentration gradients

5) Recent issues

2) Mass transport

Navier-Stokes equations for incompressible liquid

Momentum conservation

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v}.\vec{grad}\vec{v} = \eta \Delta \vec{v} - \vec{grad}P$$

Mass conservation

$$div\vec{v} = 0$$

MC Jullien's talk

2) Mass transport

Stokes equation for liquid at the nanoscale

Low Reynolds number, (quasi)-stationnary flows

$$0 = \eta \Delta \vec{v} - \overrightarrow{grad} P$$

$$div \vec{v} = 0$$

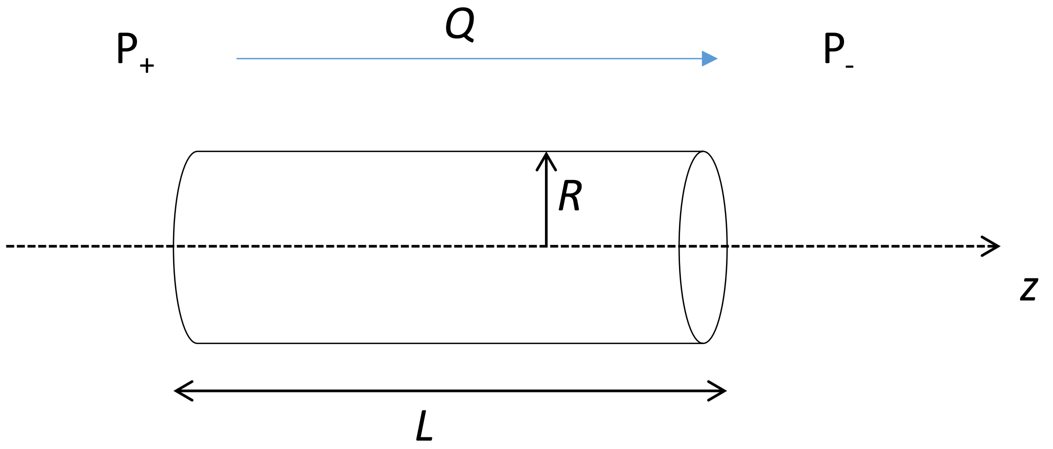
+ Boundary conditions

2) Mass transport

No-slip boundary condition : $v_s=0$

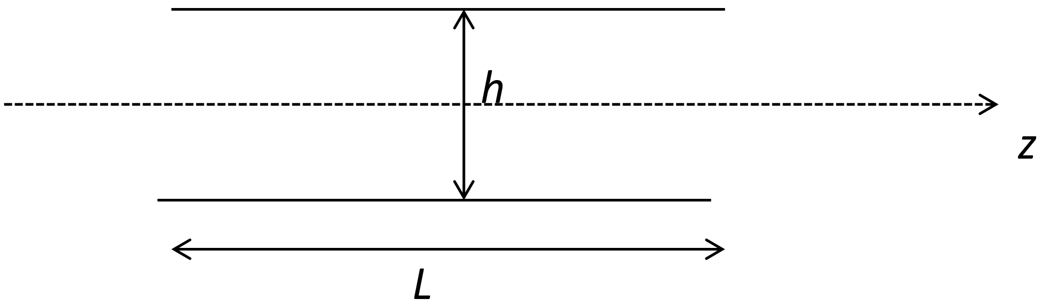
Tubes

$$Q = \frac{\pi R^4}{8\eta L}\Delta P$$



Slits

$$Q = \frac{h^3}{12\eta L}\Delta P$$



2) Mass transport

QUIZZ

The flow rate through a tube of radius R= 10 nm, length L=10 µm and under a pressure difference of 1 bar is:

a) 1 L/s
b) 1 µL/s (10^{-6} L/s)
c) 10 zL/s (10^{-20}L/s)

2) Mass transport

Measuring flow rate at the nanoscale: a difficult task!

#1 Direct measurement **inside** the channel

#2 Measurements of species transport at the **exit of the channel**

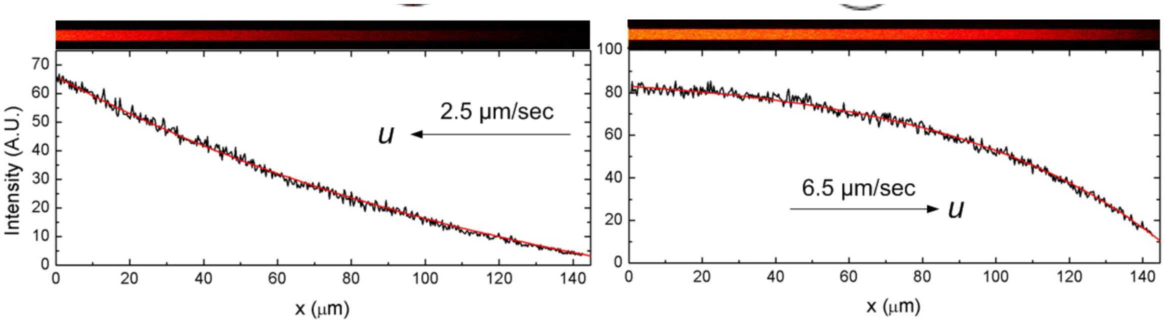
#3 Integrated measurements

2) Mass transport

Measuring flow rate at the nanoscale: a difficult task!

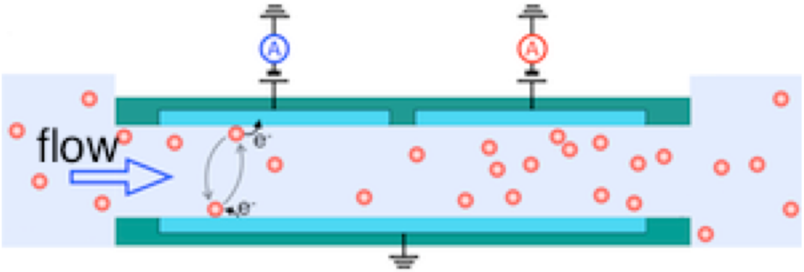
#1 Direct measurement **inside** the channel

- Diffusion/advection balance of dye repartition Q~50fL/min



Lee et al., *PRL* 2014

- Cross correlation spectroscopy Q~1pL/min

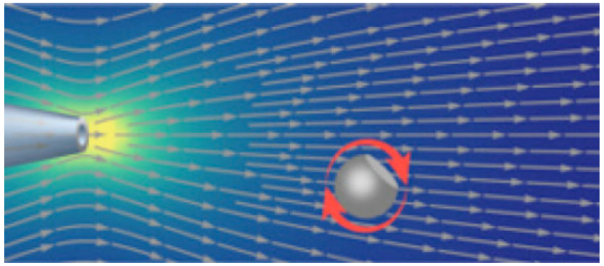


(Mathwig et al. PRL 2012) –
no need of optical acess.

2) Mass transport

#2 Measurements of species transport at the **exit of the channel**

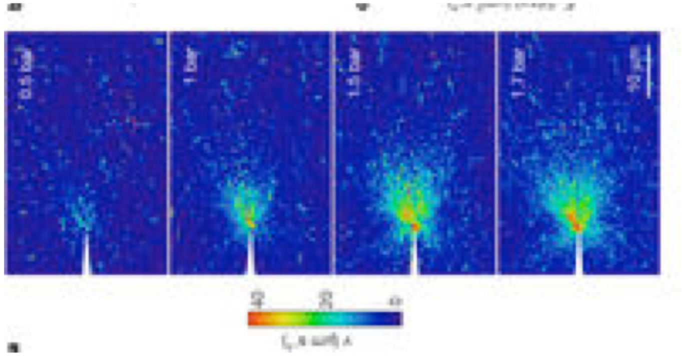
- Landau-squire jet
  - *Punctual source, momentum conservation*



○ Probed with optical tweezers **:** Q ~ 100 pL/min

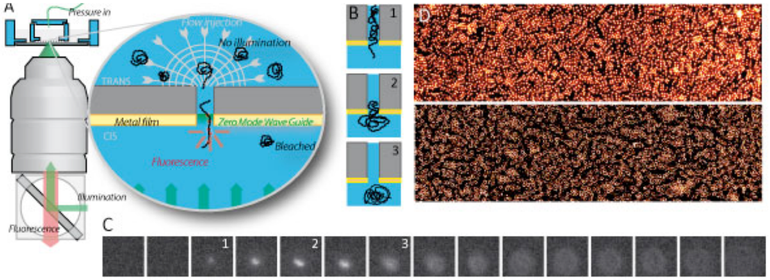
(Laohakunakorn et al., Nanoletters 2013)-

○ Fluorescent probe
(Secchi et al., Nature 2016)



- Particle translocation

○ Zero mode wave guide
(Auger et al. PRL 2014)

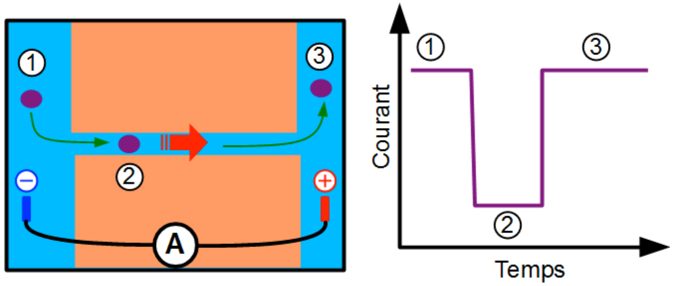


2) Mass transport

#3 Integrated measurements

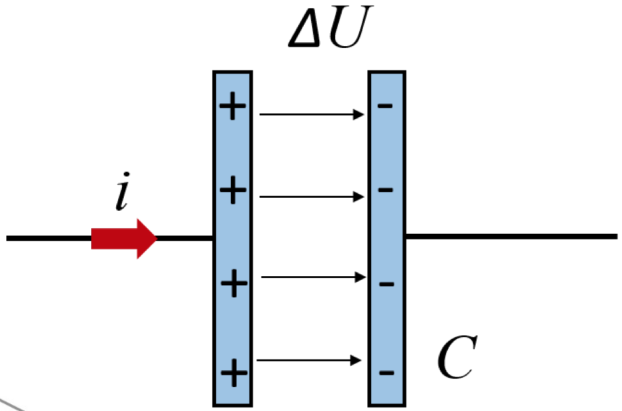
- Coulter counting 100 fL/s

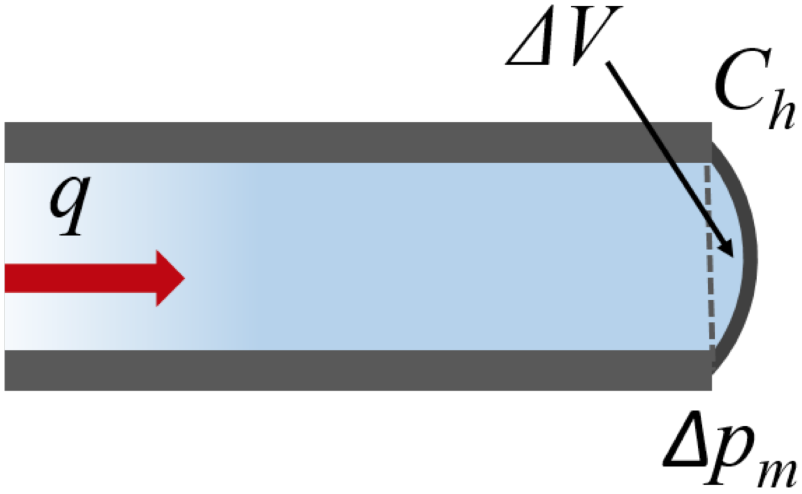
Gadaleta et al., Nanotechnology 2015



- Capacitive flow rate sensor 100 fL/s

Sharma PhD Liphy 2017





2) Mass transport

QUIZZ

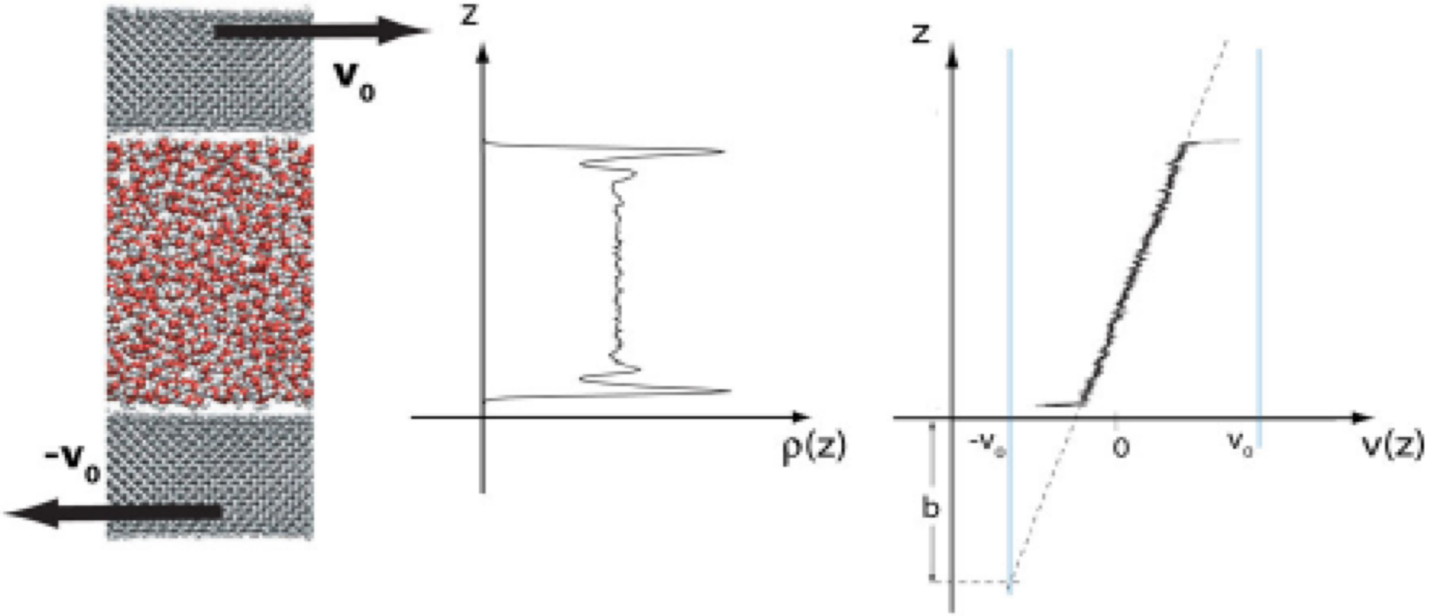
1) Who wins?

2) Who fits expectations?

2) Mass transport

Boundary condition: $v_s \neq 0$

Numerical observations



$$v(z=0) \neq v_0$$

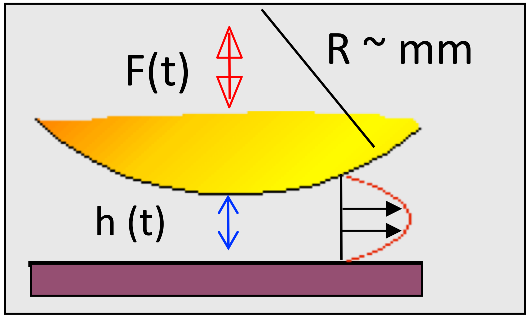
Huang et al.PRL 2008

2) Mass transport

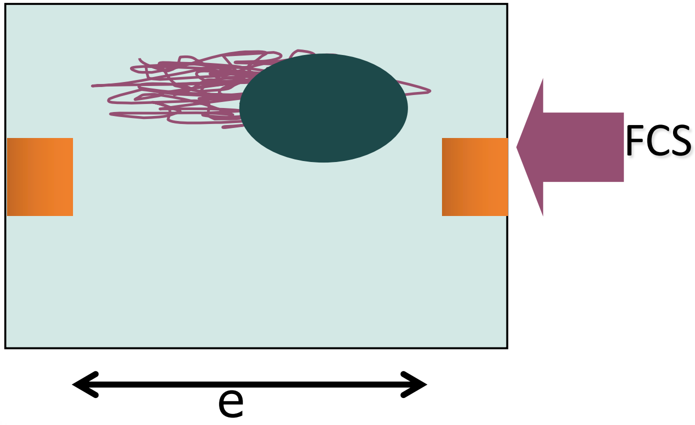
Boundary condition: $v_s \neq 0$

Experimental observations

- Surface force apparatus: *Cottin-Bizonne et al. (2005)*



- Confined Brownian movement: *Joly et al. (2006)*



2) Mass transport

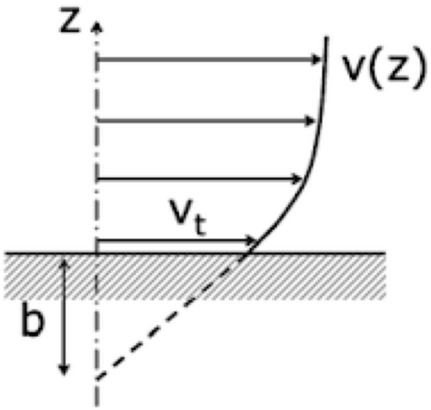
Formalism: friction coefficient and slip length

Viscous stress

$$-\lambda v_T = \eta \frac{\partial v}{\partial z}$$

$$\boxed{b = \frac{\eta}{\lambda}}$$

$$b\left(\frac{\partial v}{\partial z}\right)_{z=0} = v_t$$

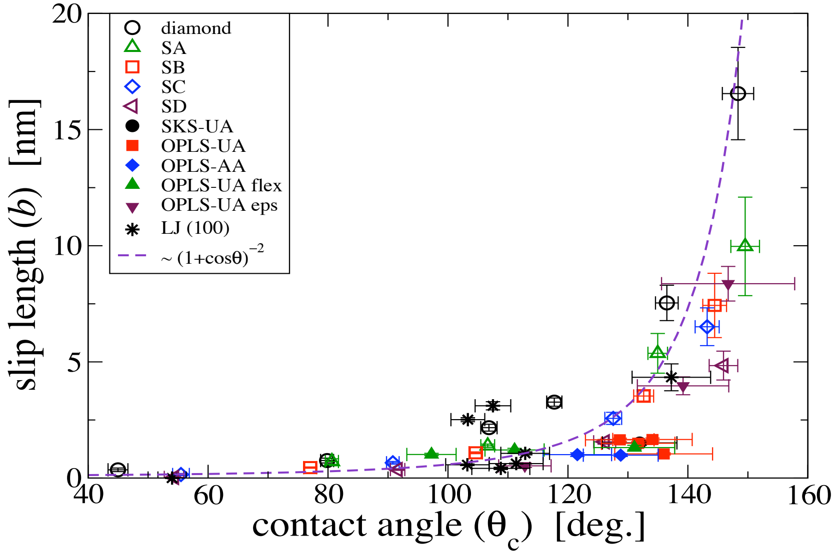


Navier 1823

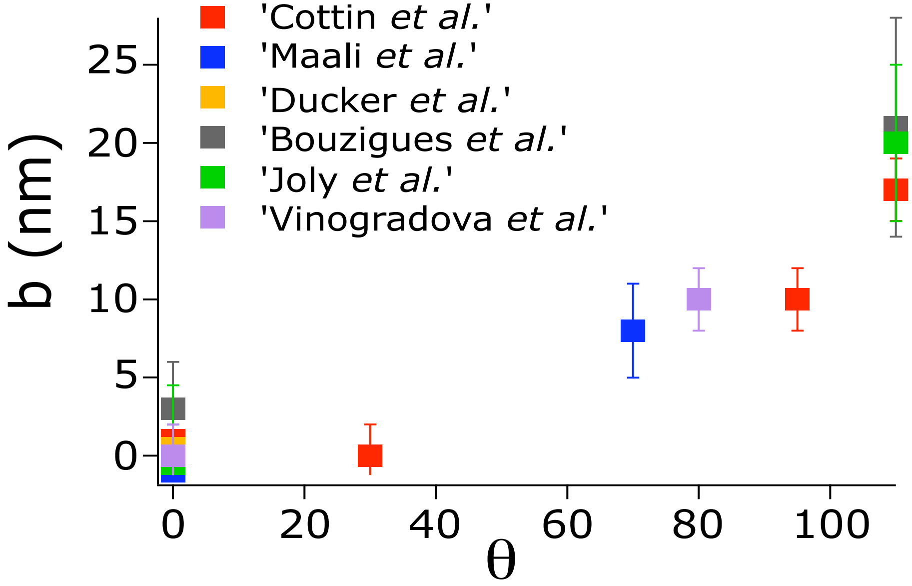
2) Mass transport

Formalism: slip length

Effect of wettability



Huang *et al.* PRL - (2008)



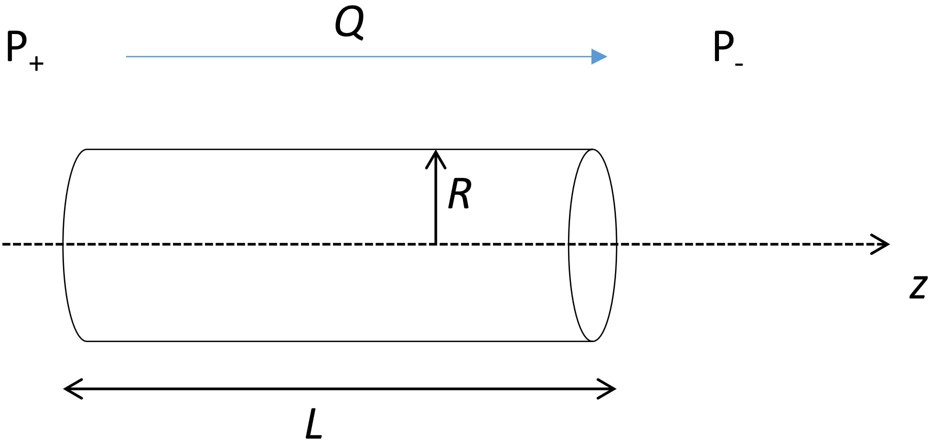
$$b \approx \frac{b_0}{(1+\cos\theta_c)^2}$$

2) Mass transport

Slip length b

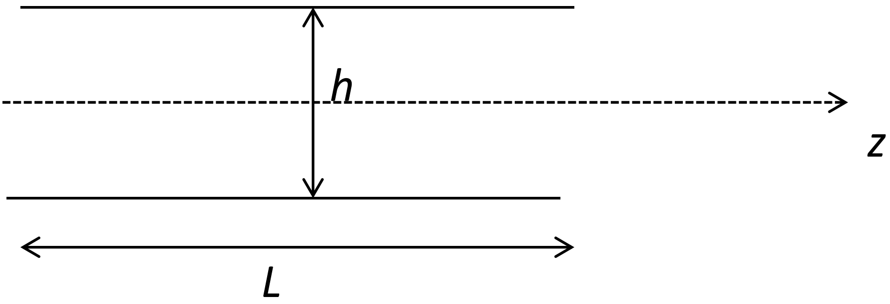
Tubes

$$Q = \frac{\pi R^4}{8\eta L}\Delta P\left(1 + 4\frac{b}{R}\right)$$



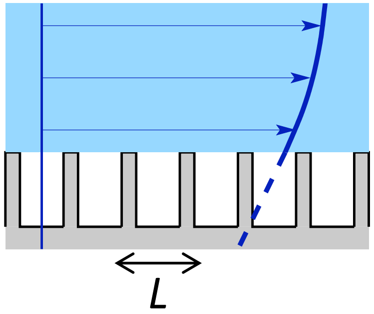
Slits

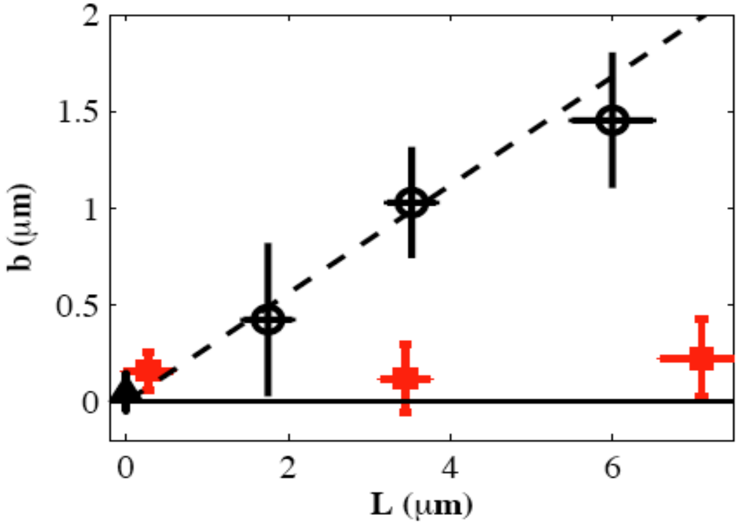
$$Q = \frac{\Delta P}{12\eta L}h^3\left(1 + \frac{6b}{h}\right)$$



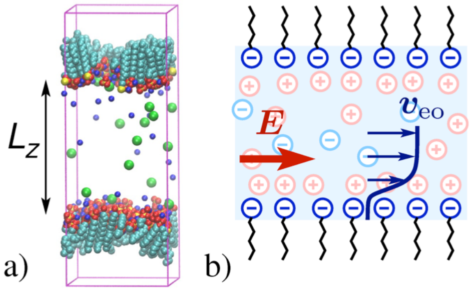
2) Mass transport

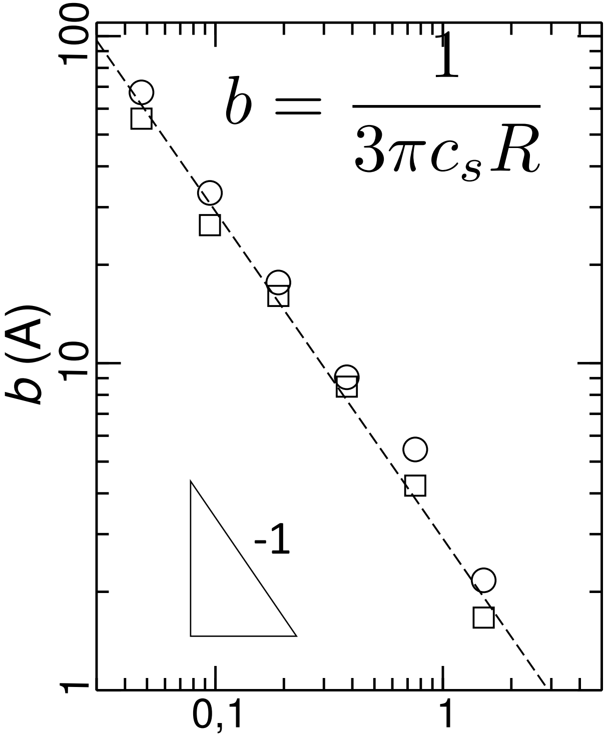
Slip length on textures





Lee, Choi, Kim, PRL (2008)

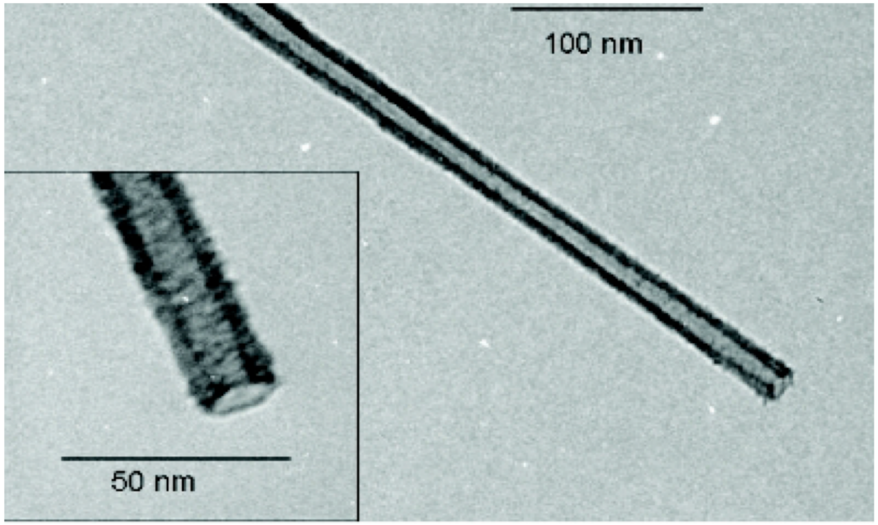


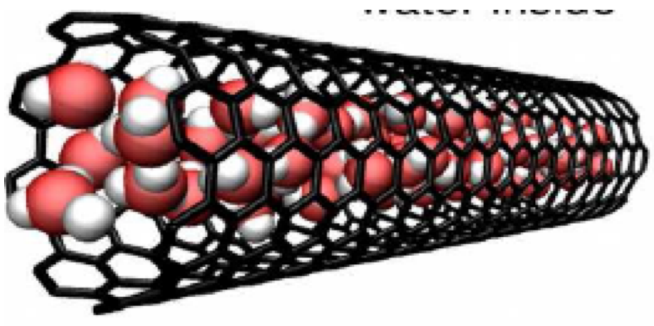


Joly et al., PRL 2014

2) Mass transport

The case of carbon nanotubes





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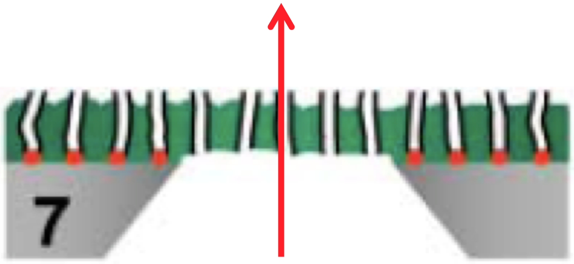
2) Mass transport

The case of carbon nanotubes

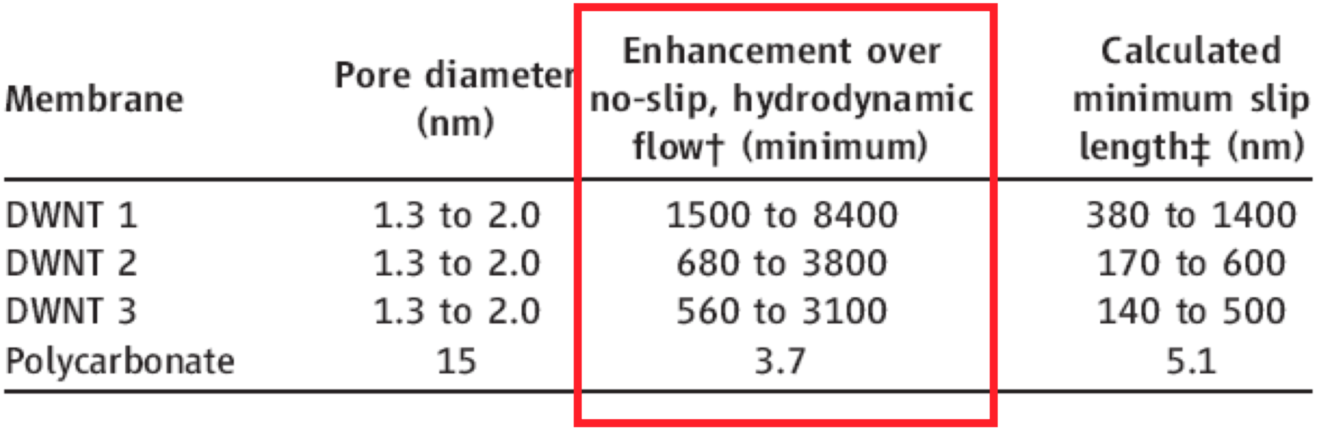
Fast Mass Transport Through Sub–2-Nanometer Carbon Nanotubes

Jason K. Holt,[1]* Hyung Gyu Park,[1,2]* Yinmin Wang,[1] Michael Stadermann,[1] Alexander B. Artyukhin,[1] Costas P. Grigoropoulos,[2] Aleksandr Noy,[1] Olgica Bakajin[1]†

Bakajin et al., Science 2006



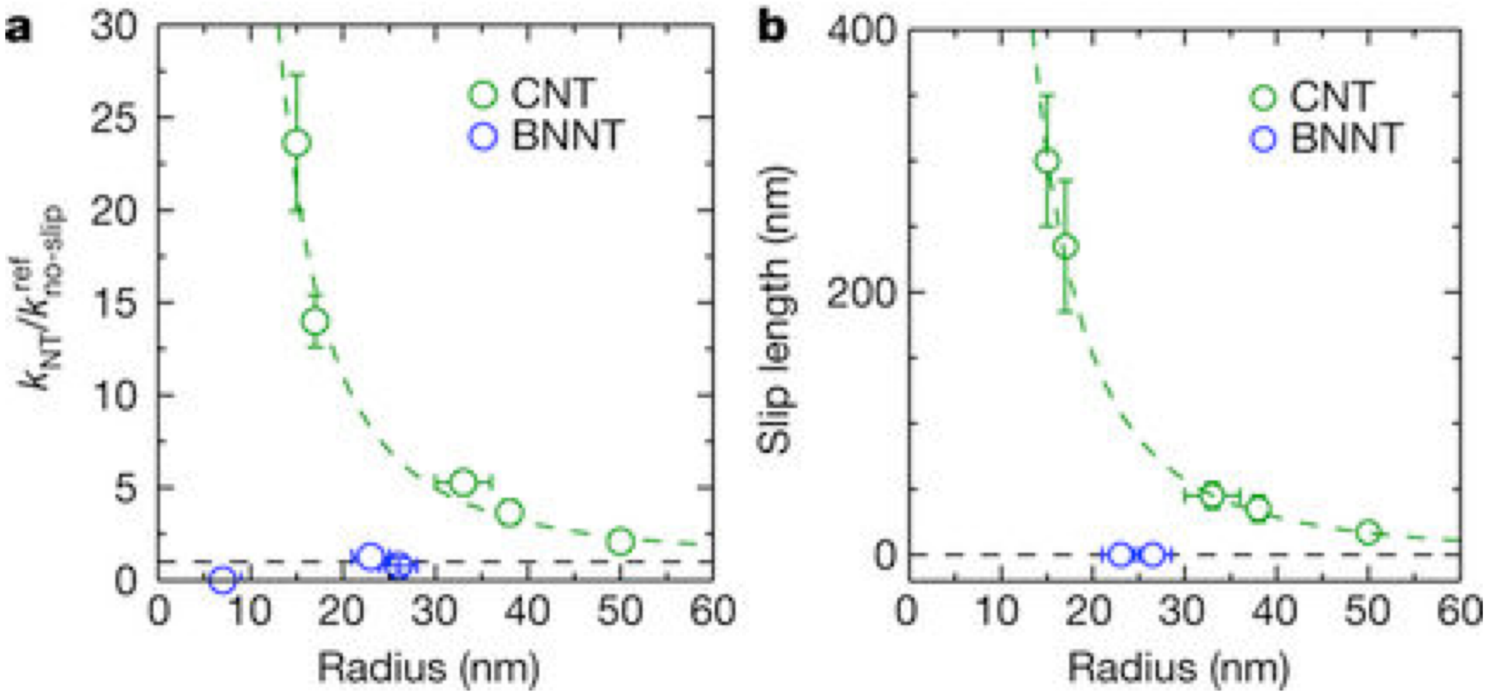
Water, …



Membrane	Pore diameter (nm)	Enhancement over no-slip, hydrodynamic flow† (minimum)	Calculated minimum slip length‡ (nm)
DWNT 1	1.3 to 2.0	1500 to 8400	380 to 1400
DWNT 2	1.3 to 2.0	680 to 3800	170 to 600
DWNT 3	1.3 to 2.0	560 to 3100	140 to 500
Polycarbonate	15	3.7	5.1

2) Mass transport

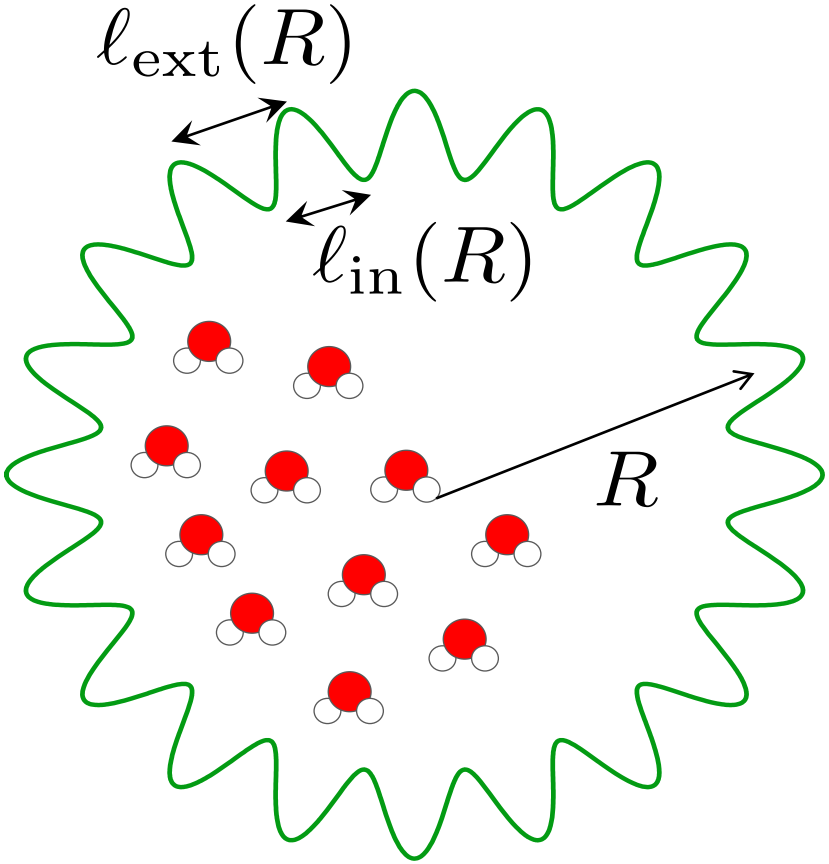
In one single tube

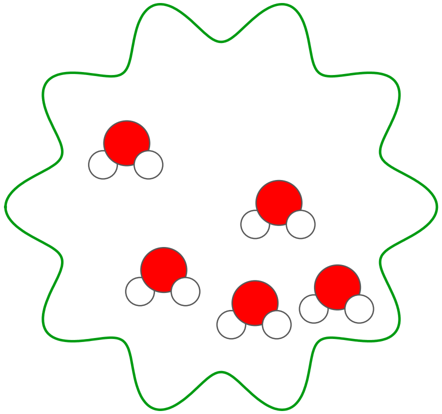


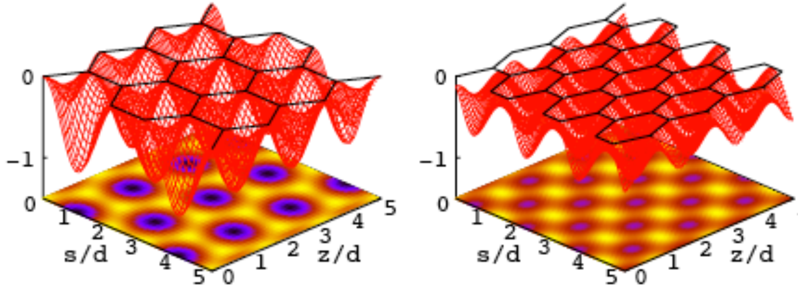
Secchi et al., Nature 2016

2) Mass transport

Commensurability and Mango effect







a) graphene

b) (10,10) CNT

Smoothing of energetic landscape graphite/water

K. Falk et al. NanoLetters (2010)

2) Mass transport

QUIZZ

1) 0rder of magnitude of the slip length – 1 or 100 nm.
    a) Near a smooth hydrophobic surface
    b) In a nanometric carbon nanotube

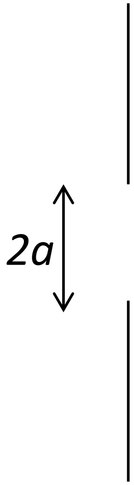
2) Does the slip length depends on the fluid velocity? Viscosity?

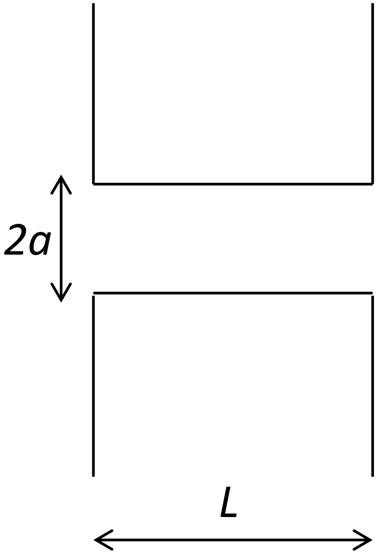
3) For CNNT, which ones are the most slippery? The large one or the small ones?

2) Mass transport

Effect of geometry

$$Q = \frac{\pi R^4}{8\eta \enclose{circle}{L}} \Delta P$$





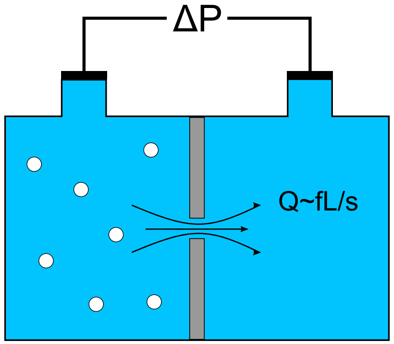
Ultrathin membranes:

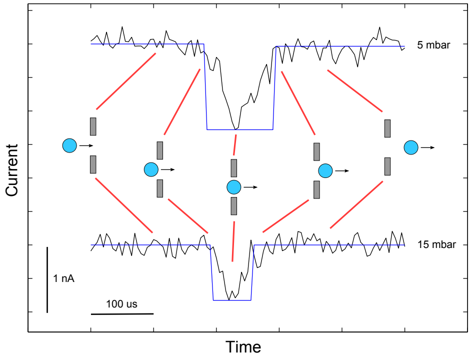
$$Q = \frac{a^3}{3\eta} \Delta P$$

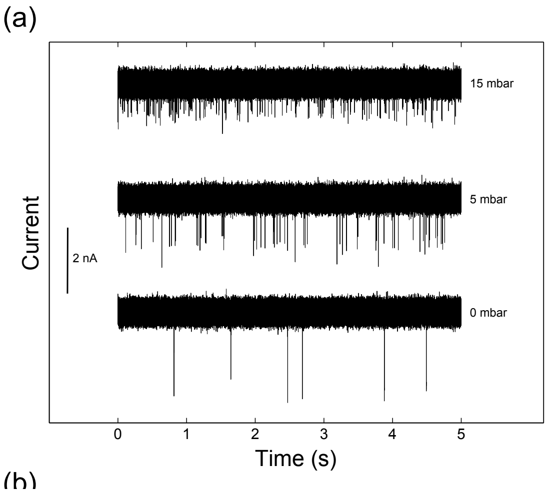
$$Q = \left(\frac{\pi R^4}{8L} \left(1 + 4\frac{b}{R} \right) + \frac{a^3}{3} \right) \frac{\Delta P}{\eta}$$

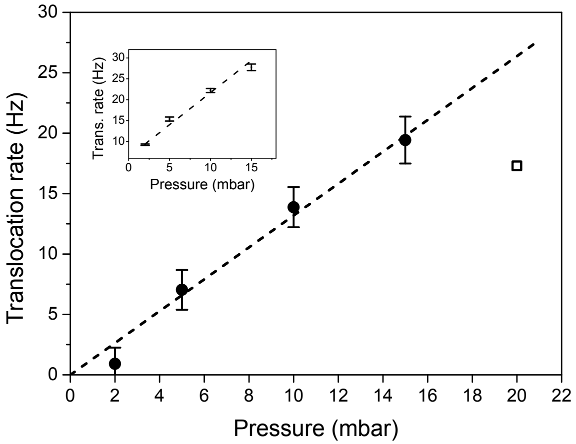
2) Mass transport

Consequence 1: Coulter counter





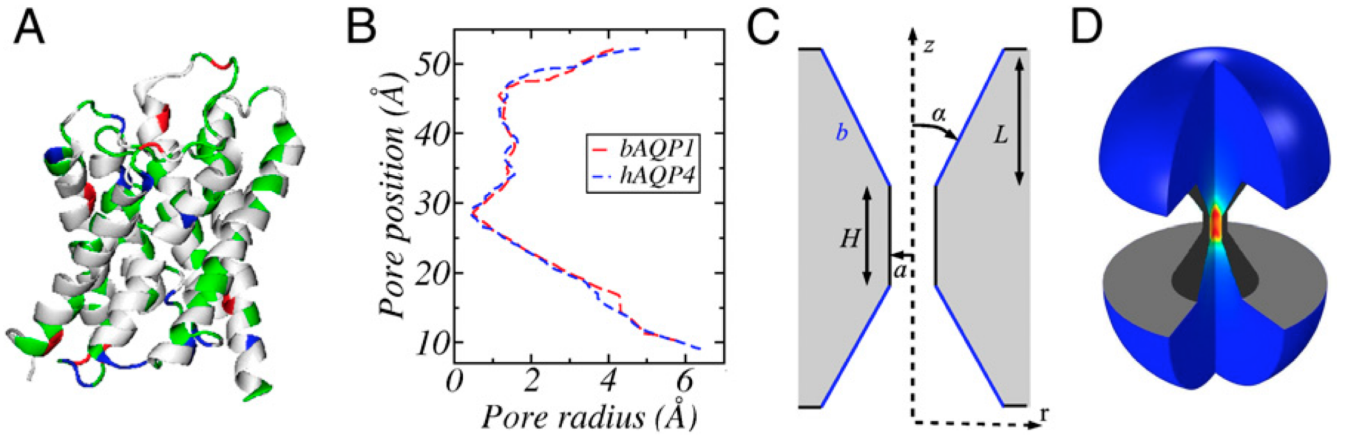


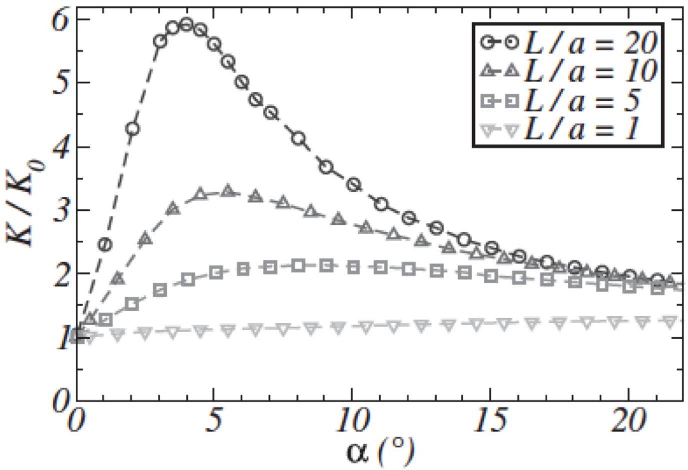


R=85 nm
L=50 nm
13.2 fL/mbar vs 13.9 fL/mbar

2) Mass transport

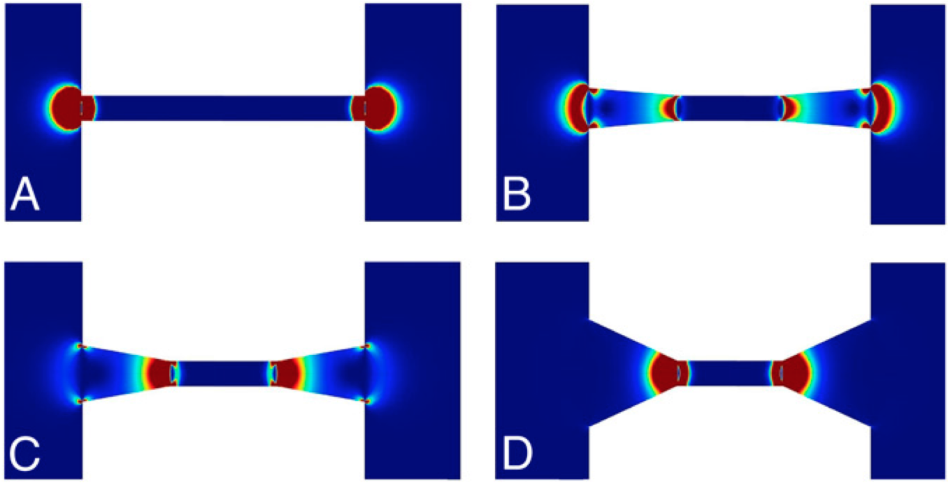
Consequence 2: the case of aquaporins

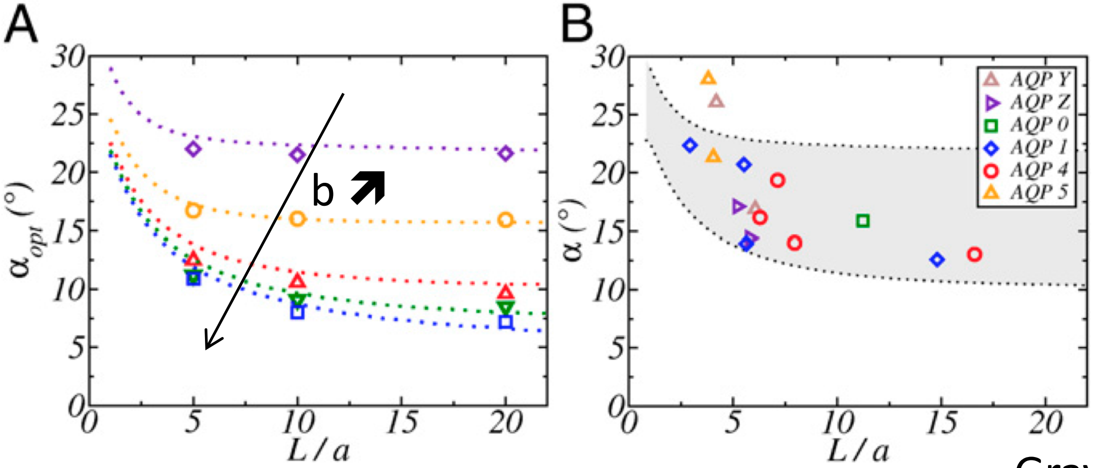




2) Mass transport

Consequence 2: the case of aquaporins

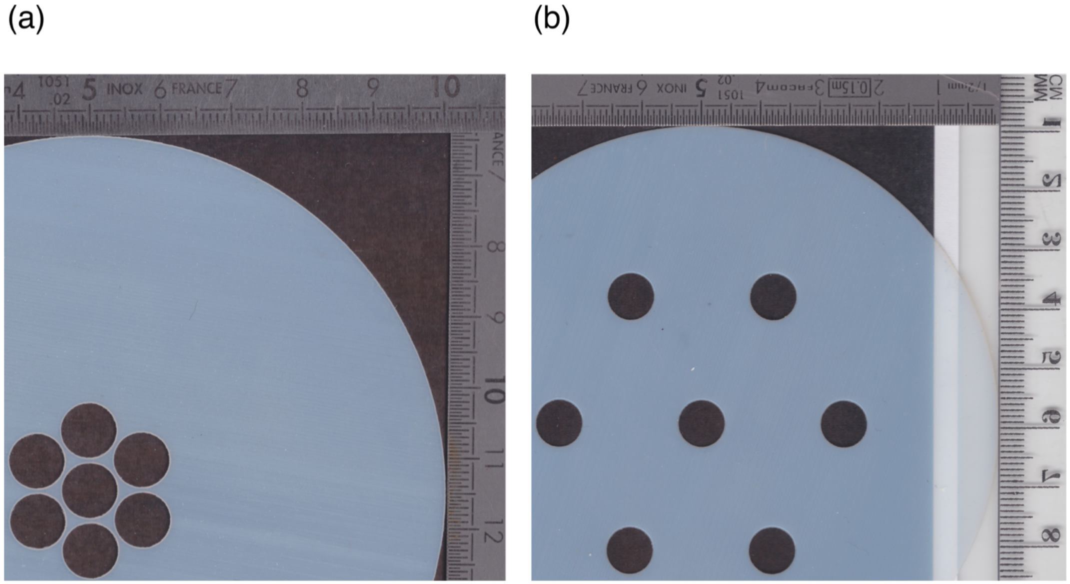




Gravelle et al., PNAS 2013

2) Mass transport

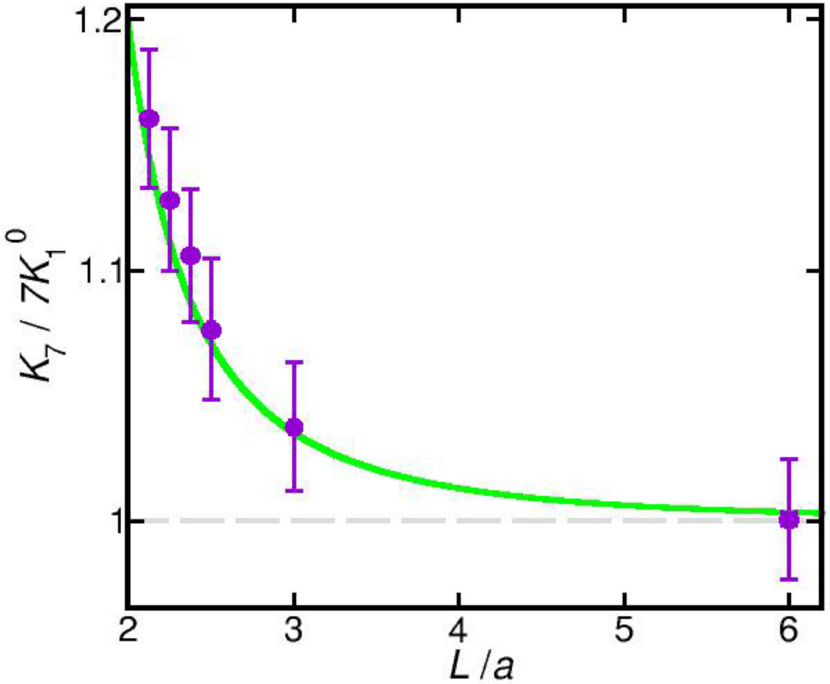
From one to many pores



PhD C. Sempere

2) Mass transport

From one to many pores, enhanced permeability



L = distance between pores

$$\frac{K_7}{7K_1^0} = \frac{1}{7}\Sigma_i\left(1 - \frac{\lambda^i\left(\frac{a}{L}\right)}{1 + \frac{8\ell}{3\pi a}}\right)^{-1}$$

$$\lambda^i = \sum_{j,j\neq i}\left[\frac{2}{3\pi}\left(\frac{a}{L_j}\right)^3 + \frac{6}{5\pi}\left(\frac{a}{L_j}\right)^5 + \frac{18}{7\pi}\left(\frac{a}{L_j}\right)^7 + \frac{56}{9\pi}\left(\frac{a}{L_j}\right)^9\right]$$

PhD C. Sempere

Content

1) Introduction

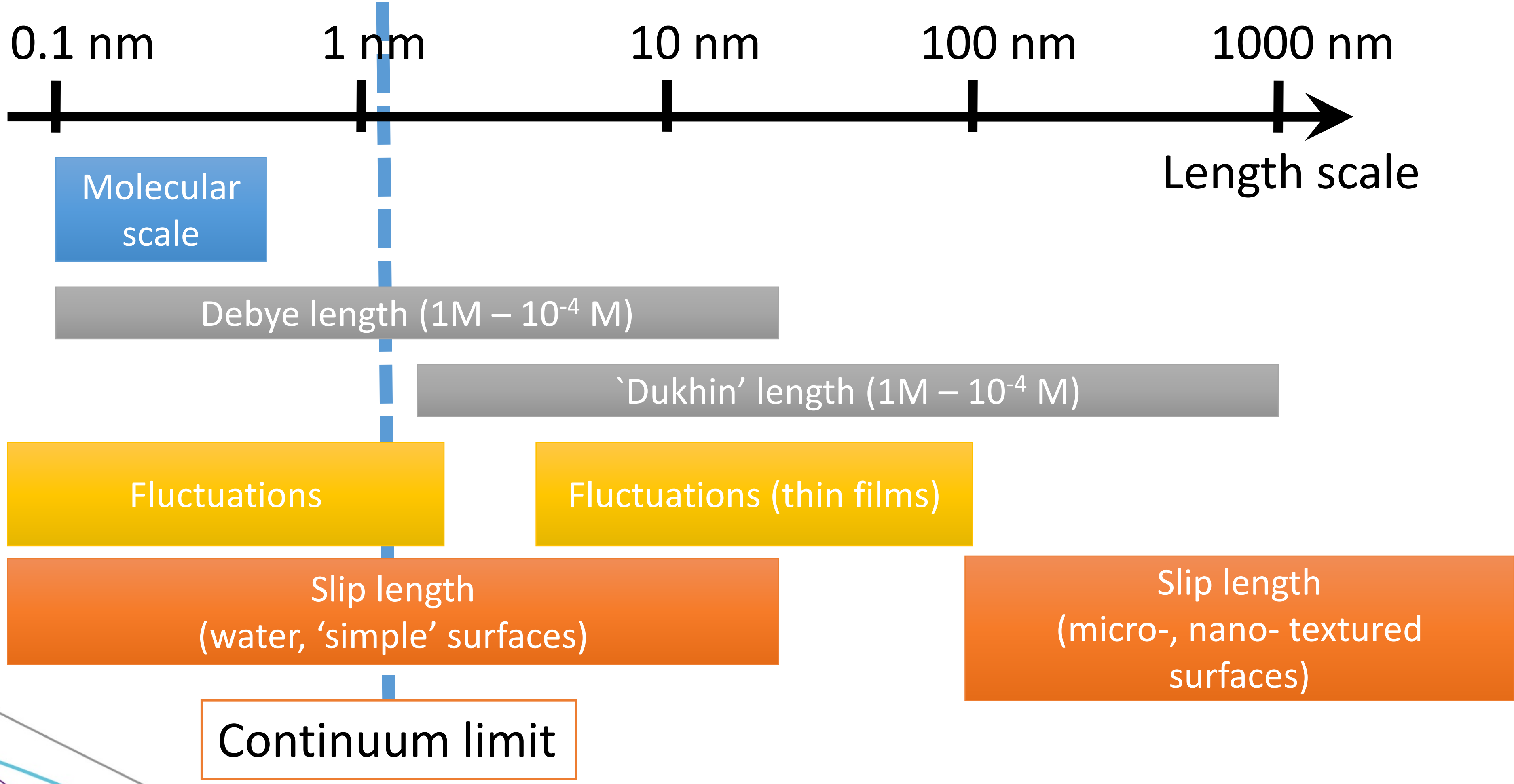
2) Mass transport

3) Electrokinetics

4) Concentration gradients

5) Recent issues

Lengthscales in fluids



0.1 nm

1 nm

10 nm

100 nm

1000 nm

Length scale

Molecular scale

Debye length (1M – 10^{-4} M)

\`Dukhin’ length (1M – 10^{-4} M)

Fluctuations

Fluctuations (thin films)

Slip length
(water, ‘simple’ surfaces)

Slip length
(micro-, nano- textured surfaces)

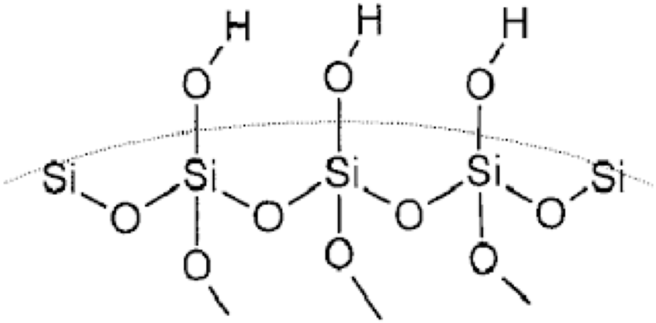
Continuum limit

Bocquet, Charlaix 2012

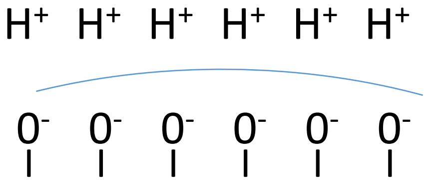
3) Electrokinetics

Coupling between electrostatics and fluid dynamics (kinetics)

- Charges at the liquid/solid interface, surface charge density noted σ



$$Si0H \rightarrow SiO^- + H^+$$

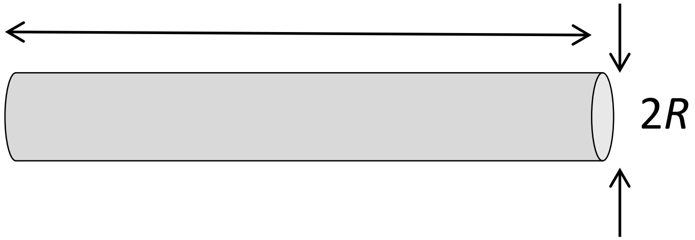


H^+ H^+ H^+ H^+ H^+ H^+

0^- 0^- 0^- 0^- 0^- 0^-

σ: number of charge per unit area, on the surface channel

Characteristic length 1



Number of bulk ions: $c_0 \pi R^2 L$

Number of surface ions: $\sigma 2\pi R L$

$$\ell_{Du} = \frac{\sigma}{c_0}$$

Dukhin length

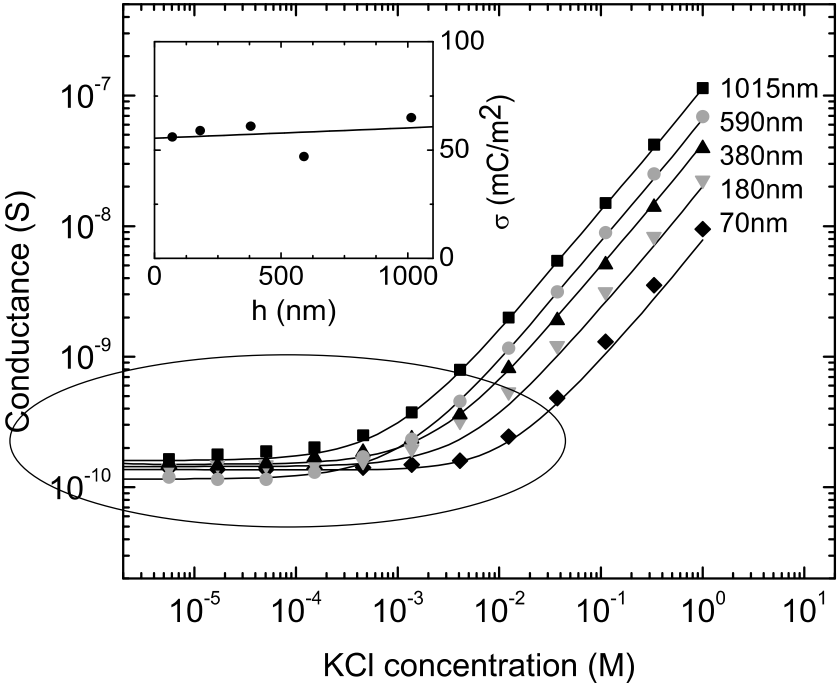
$R \gg \ell_{Du}$ bulk dominates

$R \ll \ell_{Du}$ surface dominates

From 0.1 nm to 10 µm

3) Electrokinetics

Effect on channel conductivity



Stein et al., PRL 2004

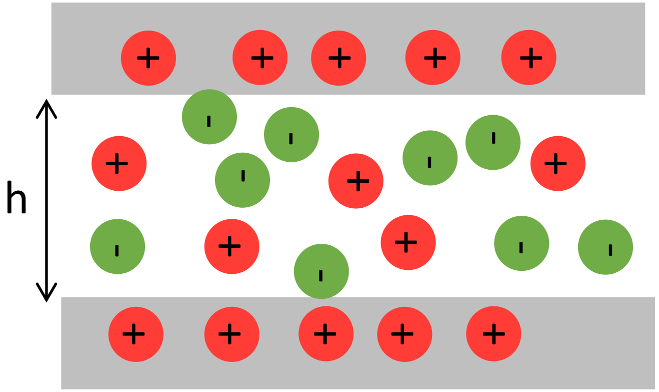
$$I = \Delta V \frac{S}{L} 2\mu c$$

$$S = wh$$

- ➢ Saturation of the conductivity at low concentration

3) Electrokinetics

Effect on channel conductivity



$$I = \int_S e(c_+ v_+ - c_- v_-)dS$$

$$v_\pm = v \pm e\mu_\pm \frac{\Delta V}{L}$$

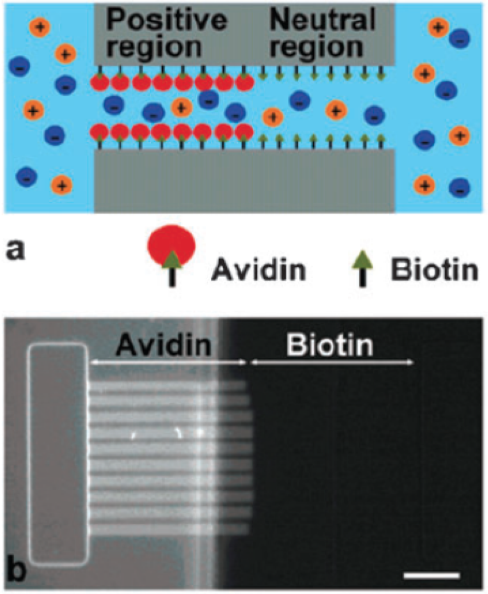
$$I = e\left((\mu_+ + \mu_-)c + K_{surf}\right) h \frac{\Delta V}{L}$$

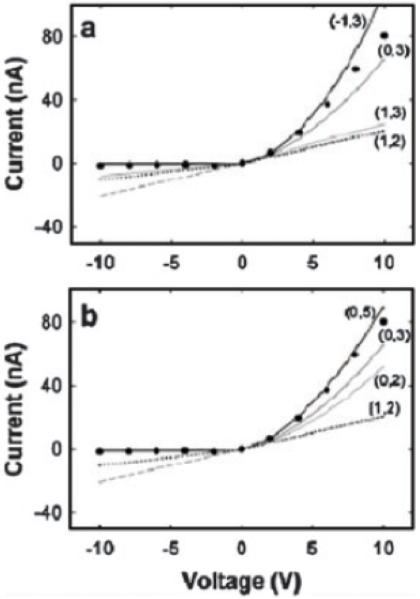
$$K_{surf} \sim K_{bulk} \times \frac{\ell_{Du}}{h}$$

Surface contribtuion

3) Electrokinetics

Effect on channel conductivity





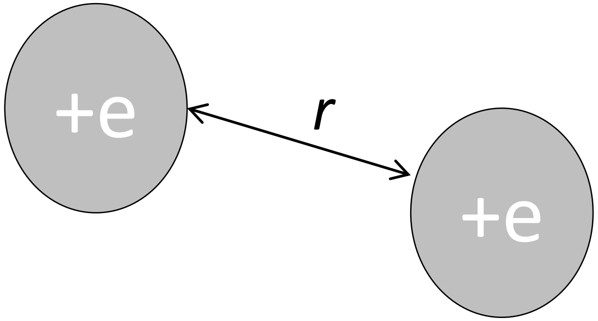
Karnik et al., Nanoletters 2007

Characteristic lengths 2-3

- Between two ions in bulk: the Bjerrum length
  Electrostatic interaction / thermal agitation

$$\ell_B = \frac{e^2}{4\pi\epsilon kT}$$

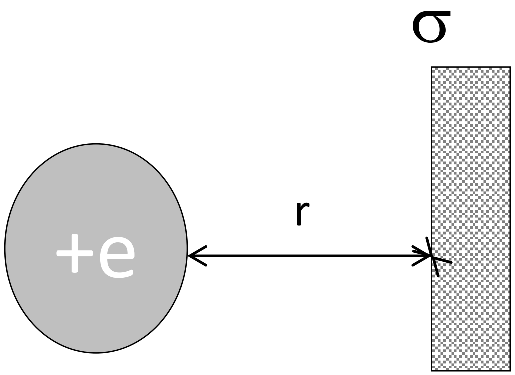
0.7 nm at 25°C



- Between counter-ions and surfaces

$$\ell_{GC} = \frac{2\epsilon kT}{e\sigma}$$

If σ~1 e^-/nm^2 -,
0.23 nm

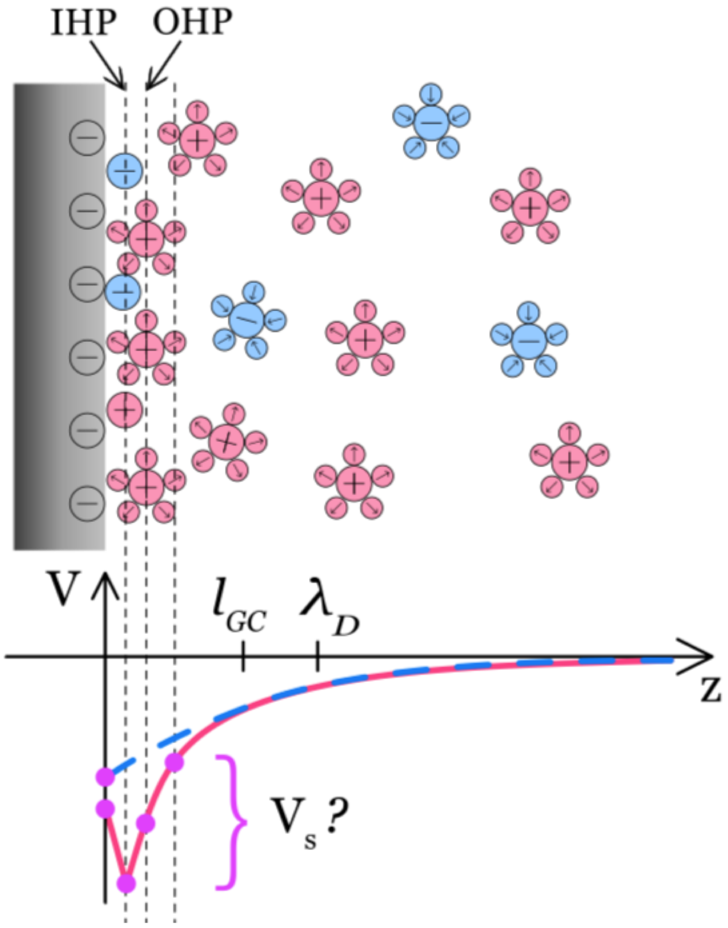


Characteristic lengths 4

- Screening length: the Debye length λ_D

$$\lambda_D = \sqrt{\frac{\epsilon k T}{2 e^2 c_0}}$$

1 mol/L, 0.3 nm



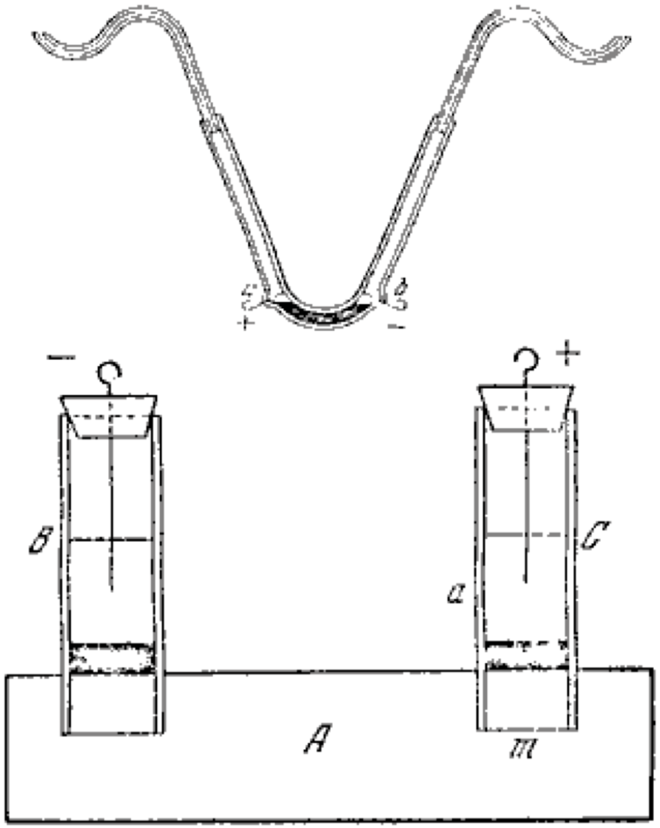
From Hartkamp et al., 2018

Coupled transport

- **Electro-osmosis**
- Streaming current
- Streaming current: energy recovery efficiency

3) Electrokinetics

Electroosmosis

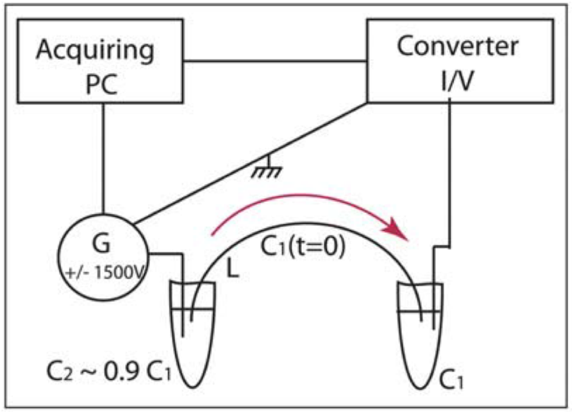


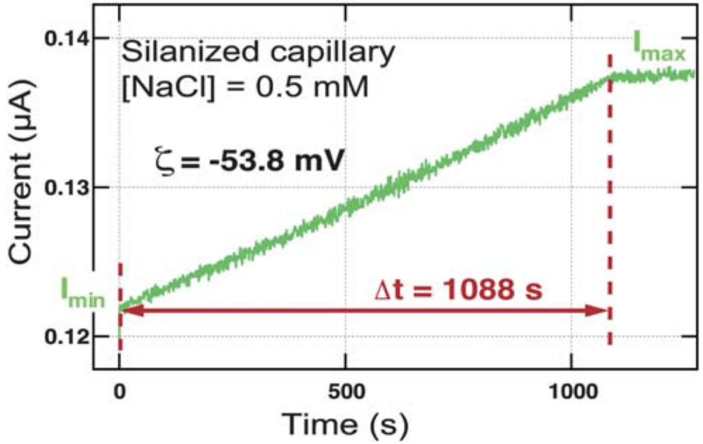
F. Reuss, Mémoires de la Société des naturalistes de Moscou, v. 2 (1809).

Applying a potential will induce a flow!

3) Electrokinetics

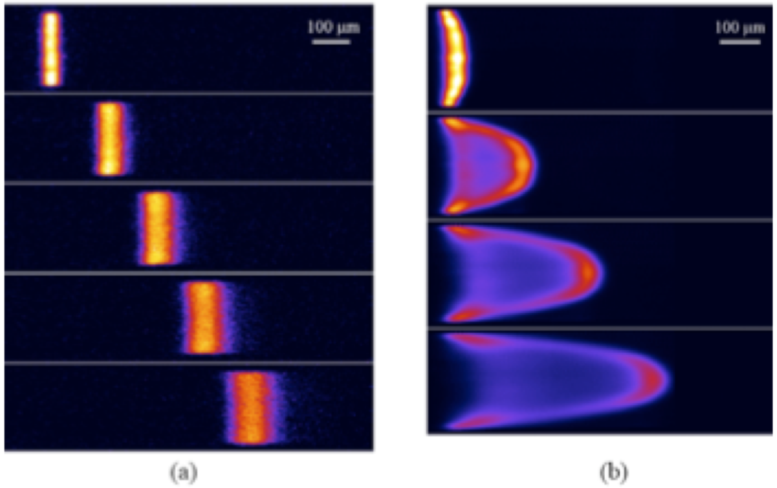
Electroosmosis





Audry 2010

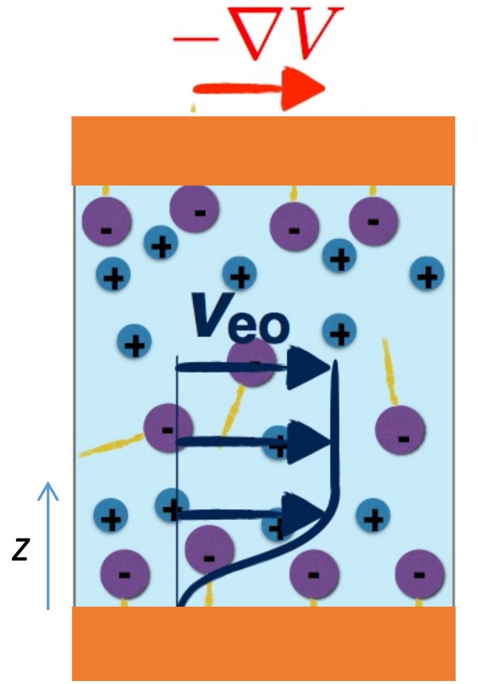
Plug flow



J.G. Santiago
Stanford Micro
Fluidics Lab.

3) Electrokinetics

Electroosmosis



- **Charges at interfaces (ionic surfactants)**
  - **Counter-ions in the vicinity of the interface**

(Electrical Double Layer: λ_D)

$$c(z) = c_0 e^{-\frac{\psi(z)}{kT}}$$

- Tangential electric field: force on the locally non-neutral liquid

$$f(z) = -(c_+(z) - c_-(z))e\nabla V$$

- Entrainment of the liquid: stationnary Stokes plug flow

$$\eta\frac{\partial^2 v}{\partial z^2} = -f(z)$$

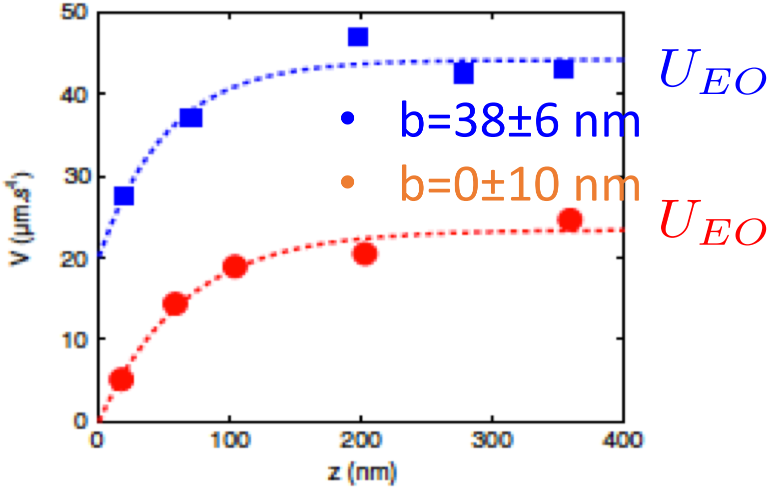
$$\boxed{v_{EO} = -\epsilon\frac{\zeta}{\eta}\nabla V}$$

Surface charge density
Hydrodynamic boundary condition

3) Electrokinetics

Electroosmosis

$$v_{EO} = -\frac{\epsilon\zeta}{\eta}\frac{\Delta V}{L}$$



Bouzigues et al., PRL 2008

No slippage

$$\zeta = V_0$$

Slippage

$$\zeta = V_0\left(1 + b\frac{V_0'}{V_0}\right) \simeq V_0\left(1 + \boxed{\frac{b}{\lambda_D}}\right)$$

No Debye overlap

Debye Hückel, low potential

Churaev 2001, Joly 2004

3) Electrokinetics

Electroosmosis / Poiseuille flow

Electroosmosis

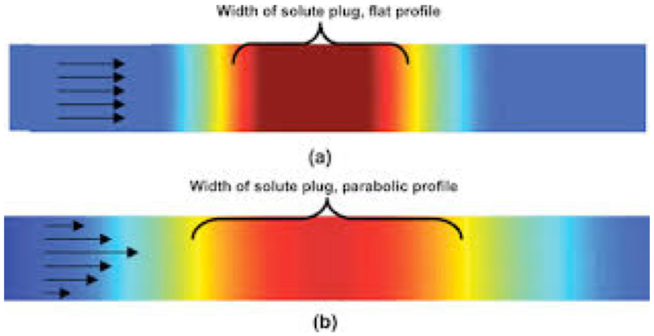
$$Q_{EO} \sim v_{EO} \times R^2$$

$$D = D$$

Poiseuille

$$Q_P \sim R^4$$

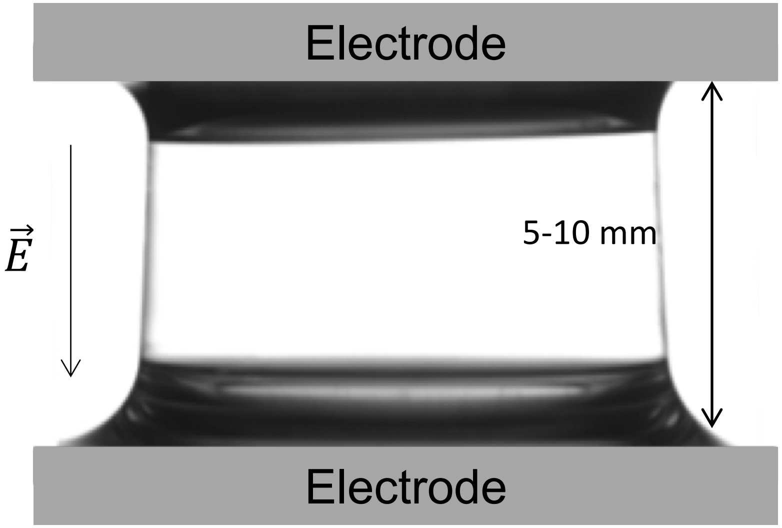
$$D_{TA} = D\left(1 + \frac{1}{192}\left(\frac{R^2 v^2}{D}\right)\right)$$

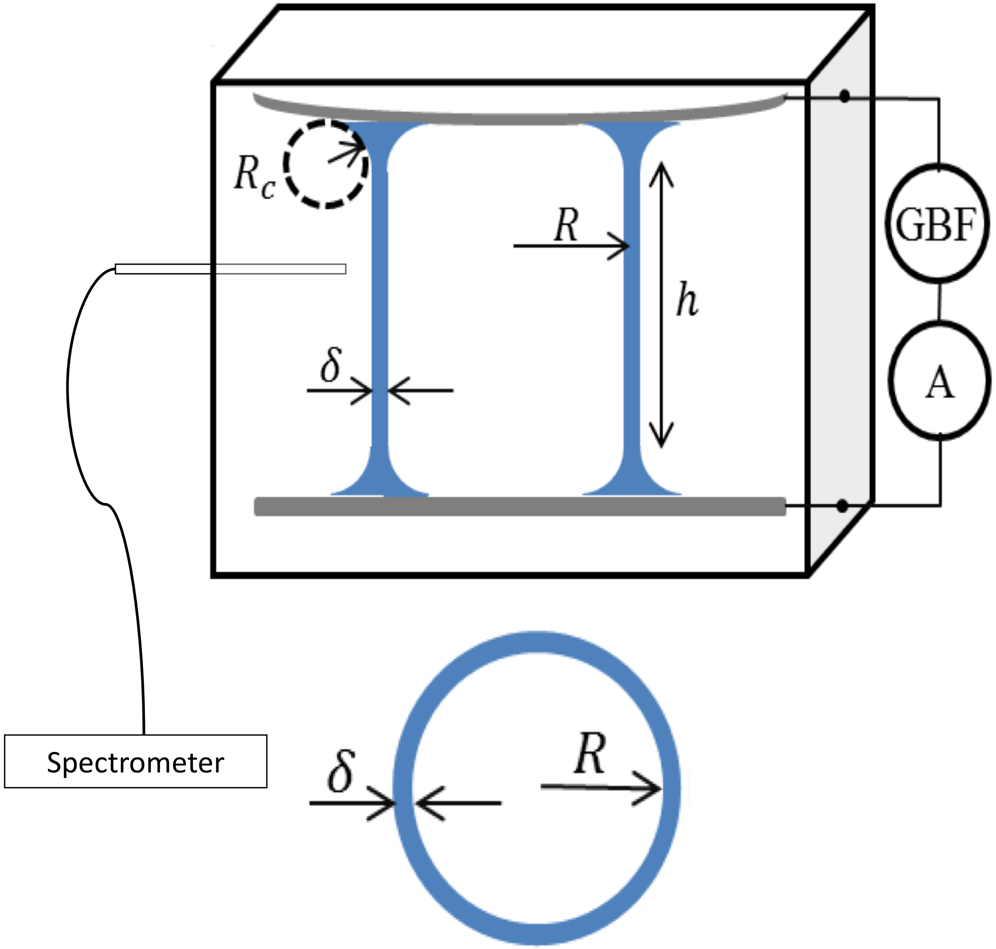


Taylor-Aris dispersion
Cf. J.-B. Salmon's talk

3) Electrokinetics

Electroosmosis in soap films



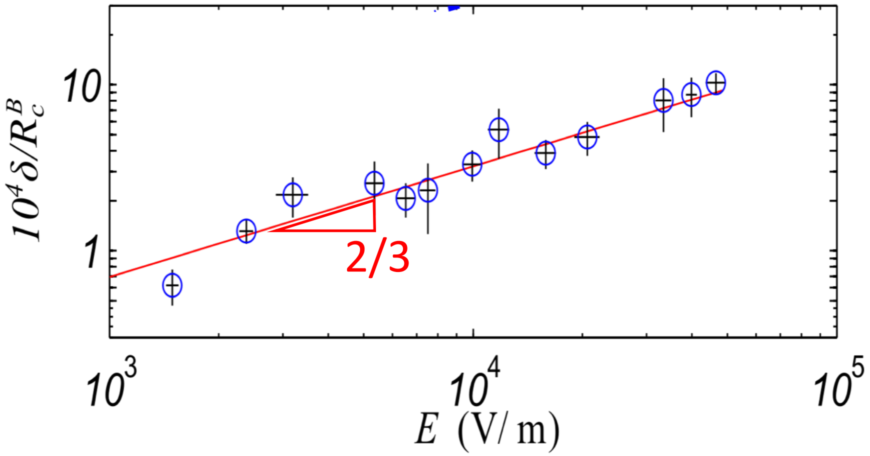


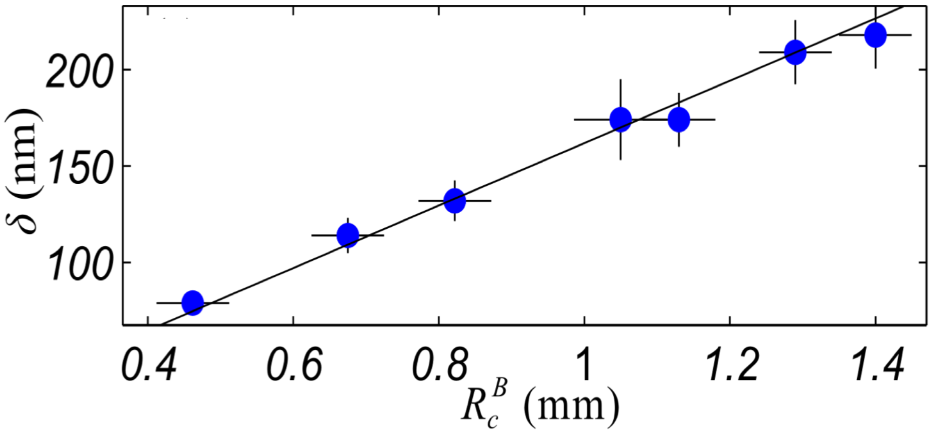
TTAB, c=cmc!

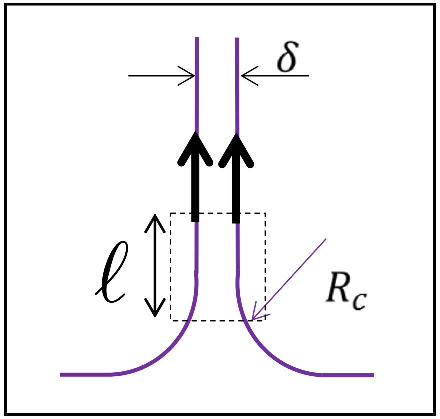
Continuous voltage

3) Electrokinetics

Electroosmosis in soap films







$$\delta \propto R_c \left(\frac{\eta v_{EO}}{\gamma}\right)^{2/3}$$

$$v_{EO} = -\epsilon\frac{\zeta}{\eta}\nabla V$$

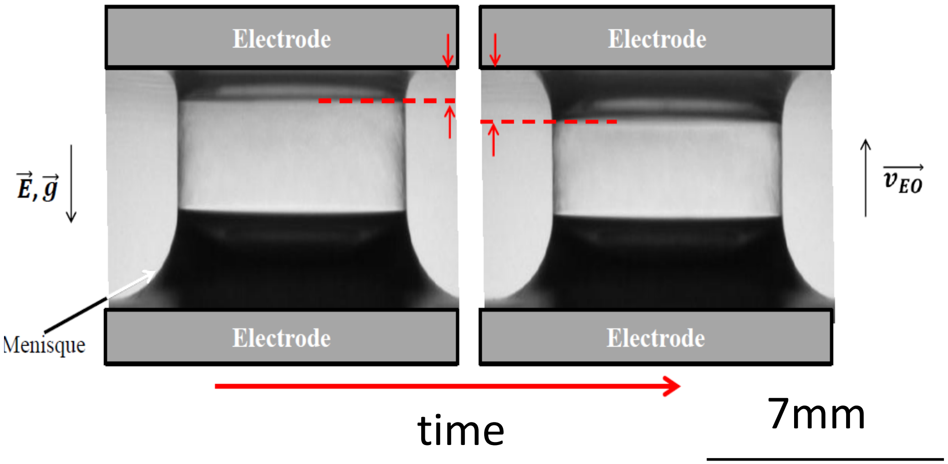
ζ=30 mV

(analogous to Landau-Levich film)

O. Bonhomme, O. Liot

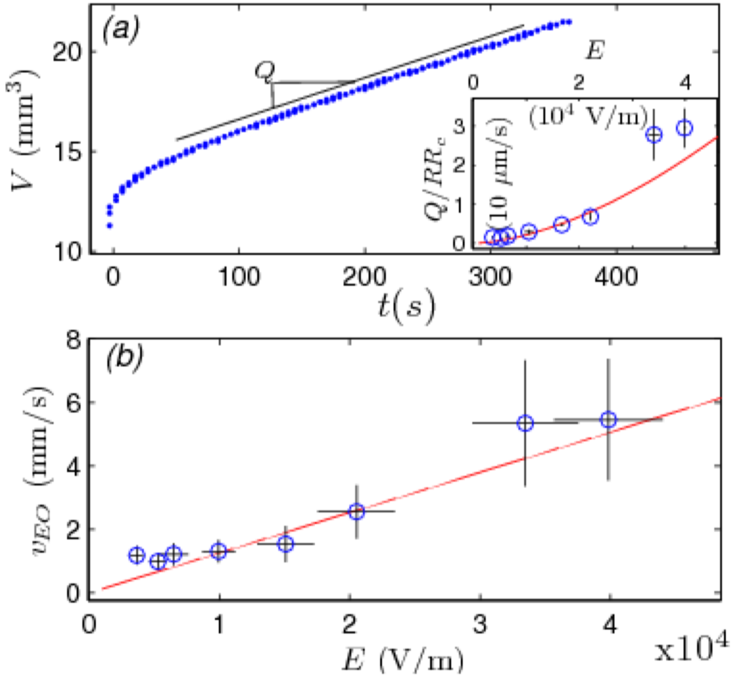
3) Electrokinetics

Electroosmosis in soap films



Bonhomme et al., PRL 2013

$$Q \sim \Delta V^{5/3}!!!$$



Good semi-quantitative results

ζ= 30 mV

QUIZZ

1) Is an electro-osmotic flow:
   a) Plug like?
   b) Parabolic like?

2) Is it affected by slippage?

3) By the surface potential?

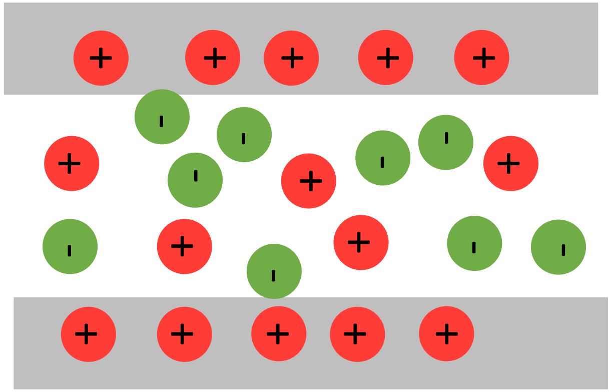
3) Electrokinetics

Coupled transport

- Electro-osmosis
- **Streaming current**
- Streaming current: energy recovery efficiency

3) Electrokinetics

Streaming current



- Velocity profile

v(r) induced by a difference of pressure (Poiseuille)

- Ion distribtuion profile near a charged surface

c(r) induced by a surface charge

$$I = \int_0^R e(c_+(r) - c_-(r))v(r)2\pi r dr$$

(tube geometry)

$$\boxed{I = -S\frac{\Delta P}{L}\epsilon\frac{\zeta}{\eta}}$$

Onsager reciprocity

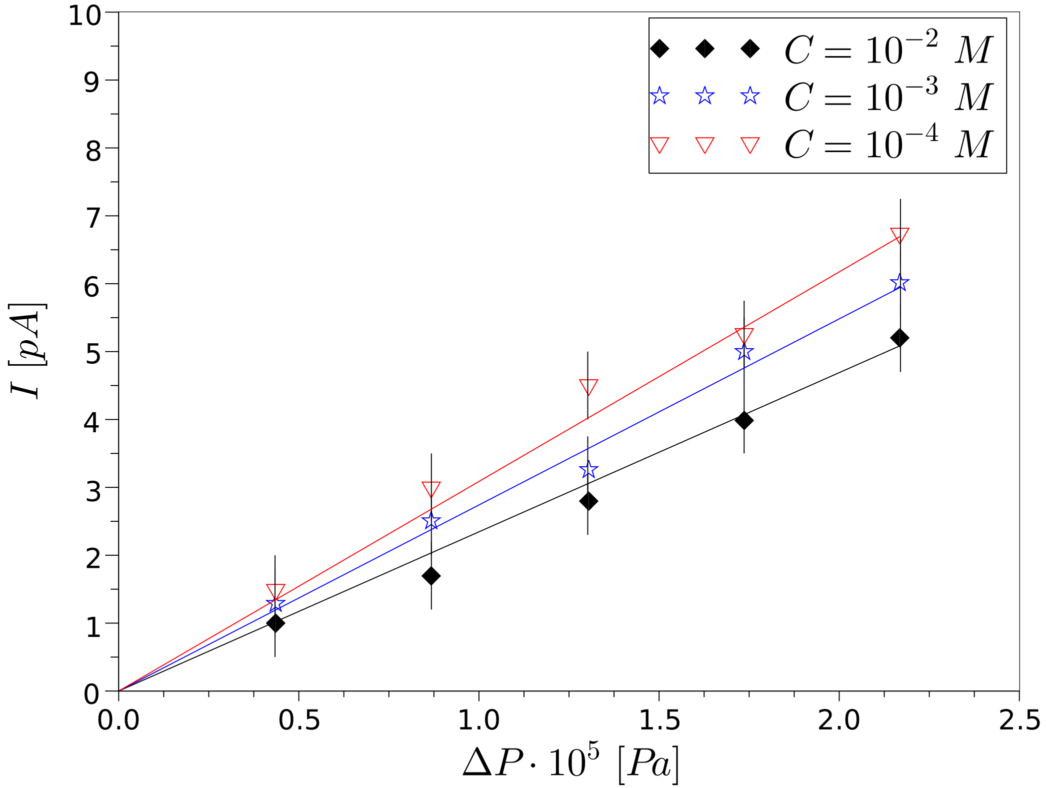
$$Q = -S\frac{\Delta V}{L}\epsilon\frac{\zeta}{\eta}$$

3) Electrokinetics

Streaming current

3) Electrokinetics

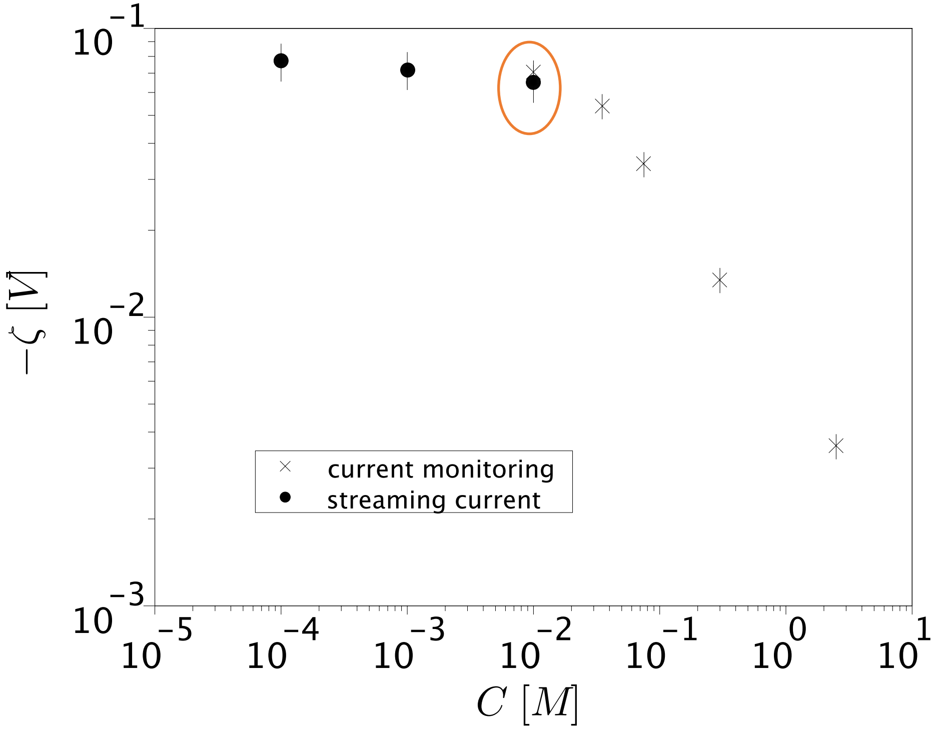
Streaming current



Glass capillary: R=1µm, L=2cm

3) Electrokinetics

Streaming current and electroosmosis



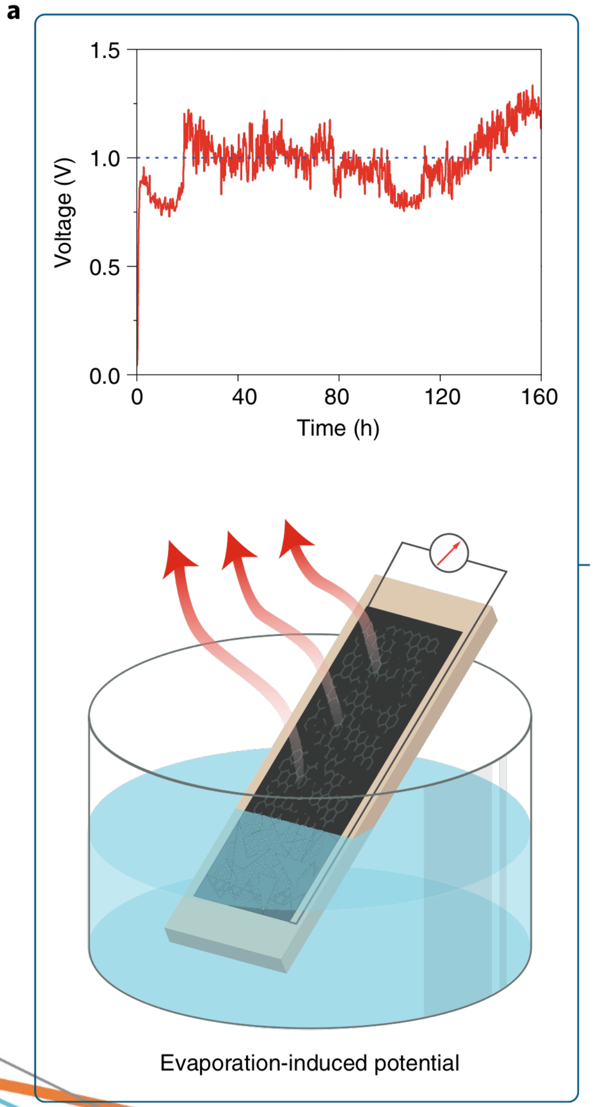
$$I = -S\frac{\Delta P}{L}\epsilon\frac{\zeta}{\eta}$$

Glass capillary: R=1µm, L=2cm

$$Q = -S\frac{\Delta V}{L}\epsilon\frac{\zeta}{\eta}$$

3) Electrokinetics

Streaming current - harvesting energy



Evaporation-induced potential

Zhang et al., Nature Nanotechnology, dec 2018

3) Electrokinetics

Coupled transport

- Electro-osmosis
- Streaming current
- **Streaming current: energy recovery efficiency**

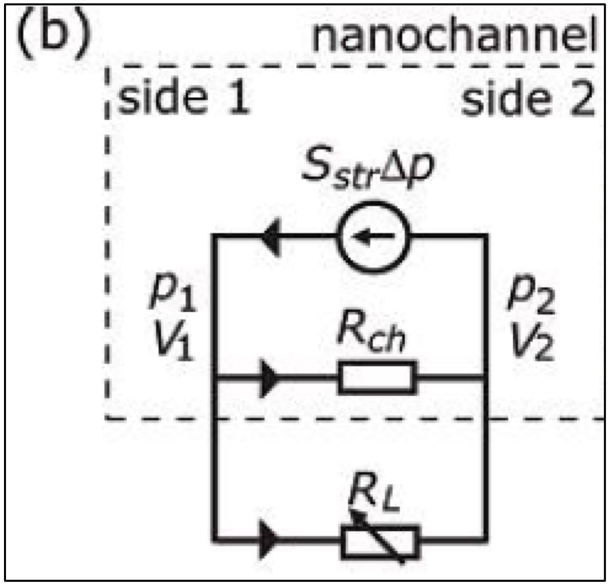
3) Electrokinetics

Streaming current: energy recovery efficiency

$$I = S_{str}\Delta p + \Delta V \frac{1}{R_{ch}}$$

$$Q = \Delta p \frac{1}{Z_{ch}} + S_{str}\Delta V$$

$$\Delta V = -S_{str}\Delta p \frac{R_{ch}R_L}{R_{ch}+R_L}$$



$$P_{out} = \frac{\Delta V^2}{R_L} \qquad P_{in} = Q\Delta p$$

$$\alpha = S_{str}^2 Z_{ch} R_{ch} \qquad k = \frac{R_L}{R_{ch}}$$

$$\epsilon = \frac{P_{out}}{P_{in}} = \frac{\alpha k}{(1+k)(1+k-\alpha k)}$$

Van der Heyden et al., Nanoletter 2006
Van der Heyden et al., Nanoletter 2007

3) Electrokinetics

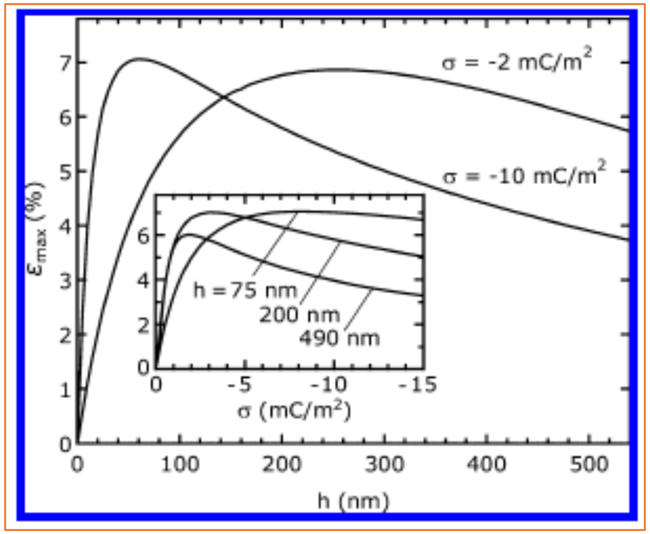
Streaming current: Energy recovery efficiency

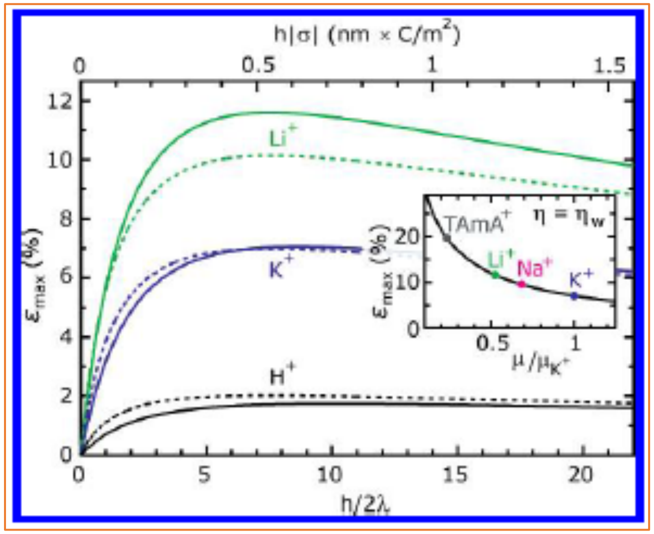
- Large surface charge density, large bulk concentration

= a lot of ions, good output (S_{str} is large)

- Charges = dissipation by conductance, bad output

(R_{ch} is small)… nanofluidic diodes?





Van der Heyden et al., Nanoletter 2006
Van der Heyden et al., Nanoletter 2007

Content

1) Introduction

2) Mass transport

3) Electrokinetics

4) Concentration gradients

5) Recent issues

4) Concentration gradients

Back to basis: osmotic pressure

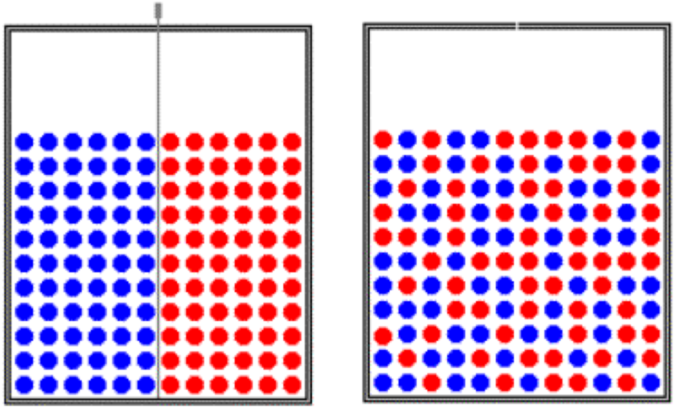
Osmotic energy
ωσμος : push

$$\Delta G = -T\Delta S$$

S = entropy in J/K
it is always positive (second principle) !

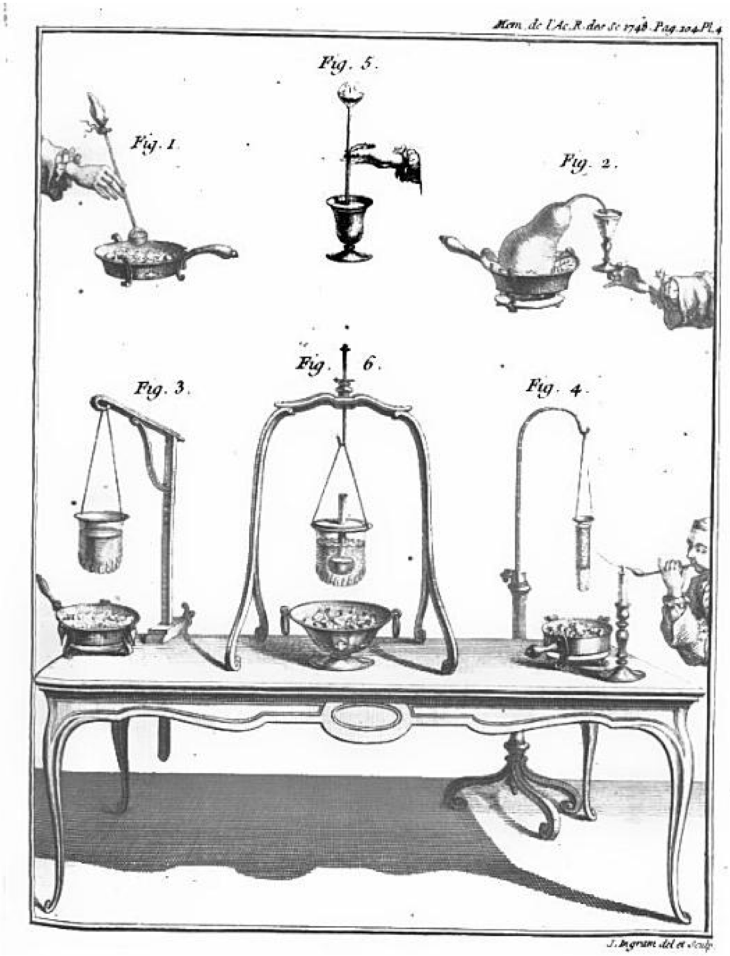
4) Concentration gradients

Back to basis: osmotic pressure



S=k ln(Ω),

Ω = numbers of states in a system



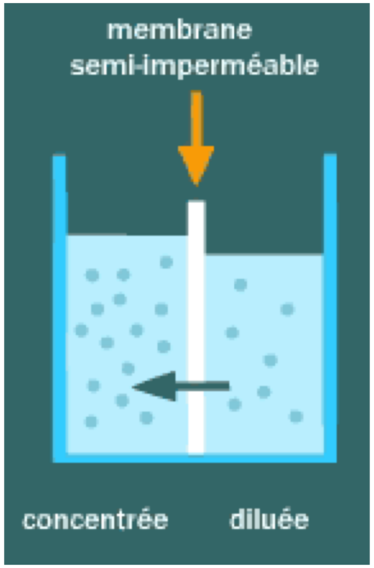
Leçons de physique expérimentale,
Abbé Nollet, 1770

4) Concentration gradients

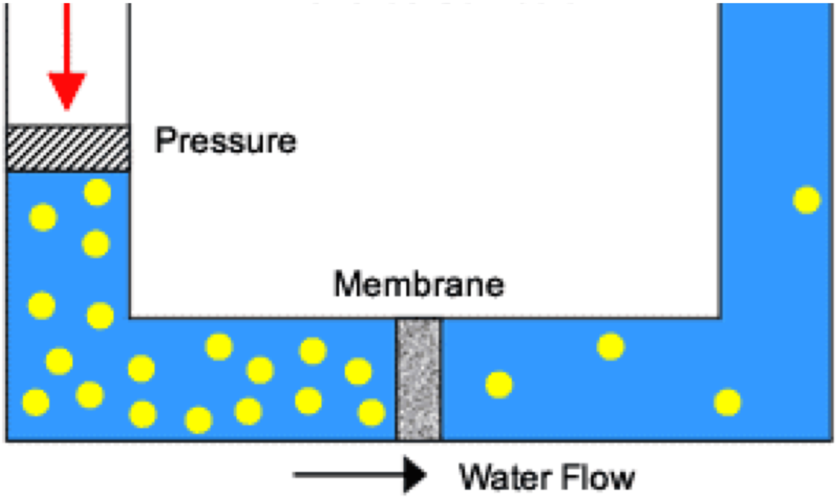
Osmotic pressure → mechanical pressure

$$\Delta\Pi = RT\,\Delta C_{\mathrm{sel}}$$

$$\Delta\Pi \approx 28\mathrm{atm}$$



280 m water fall

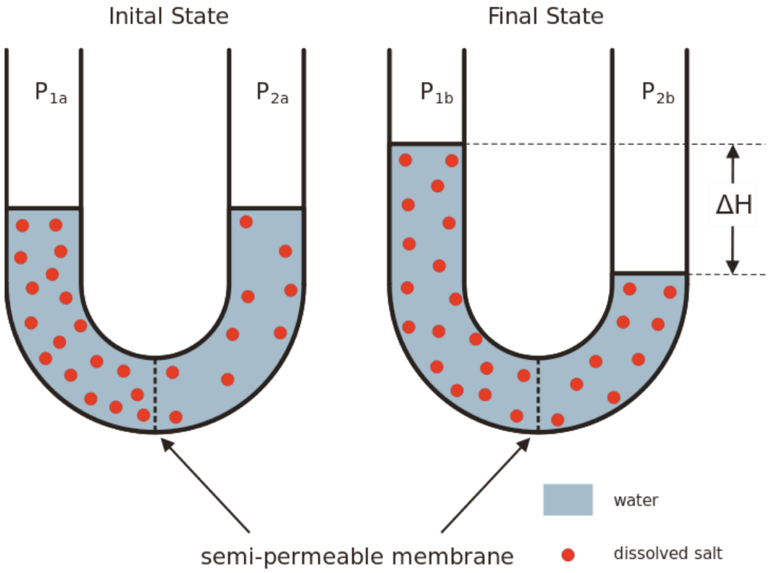


Reversing osmosis for desalination

Selective membrane

4) Concentration gradients

Osmotic pressure → mechanical pressure



$$\Delta(P - \Pi)$$

$$\Delta\Pi = 2k_B T \Delta c$$

$$P_{out} = Q\Delta\Pi \qquad Q \sim R^4!!!$$

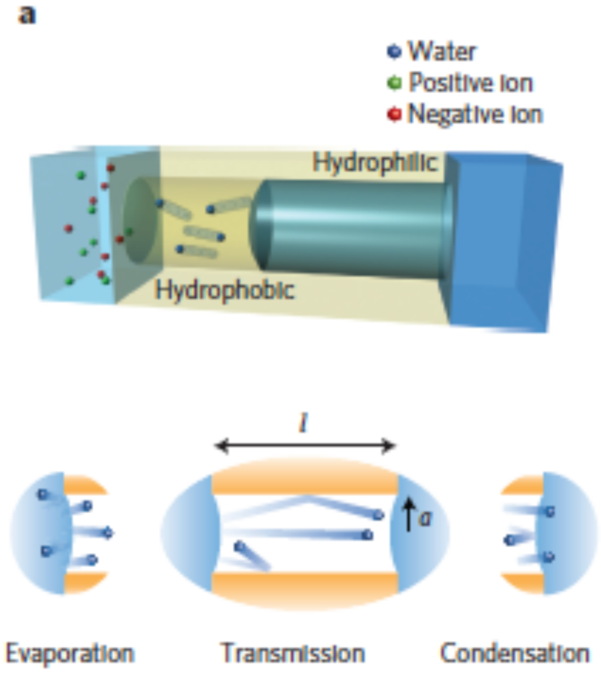
Applications: PRO (Pressure Retarded Osmosis)

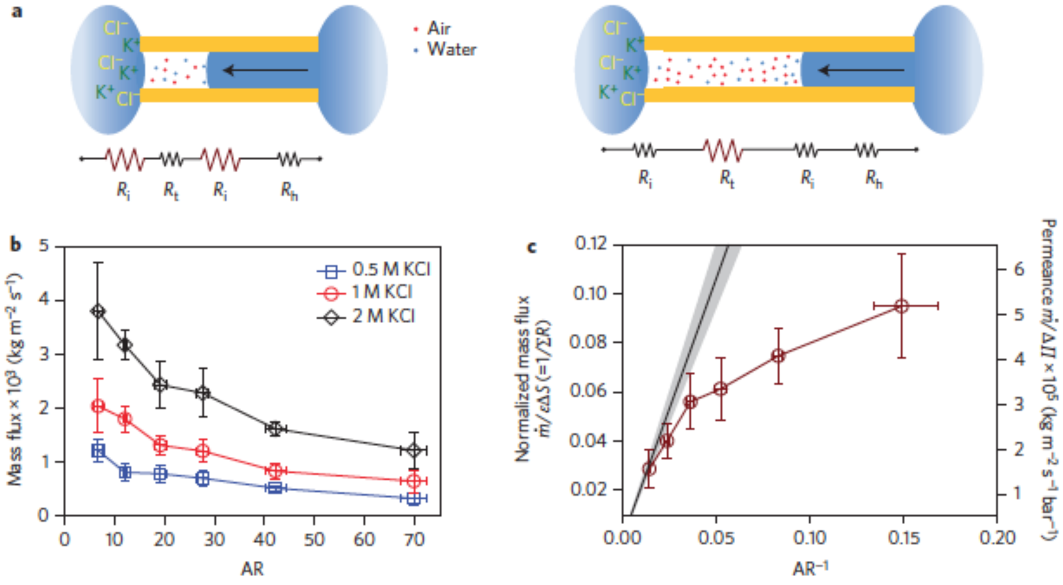
- Pore size: <0.6nm
- Power density: 1-2.7 W/m^2
- Maximal expected power: 7-8 W/m^2

4) Concentration gradients

Osmotic pressure

Selective membranes: new strategies





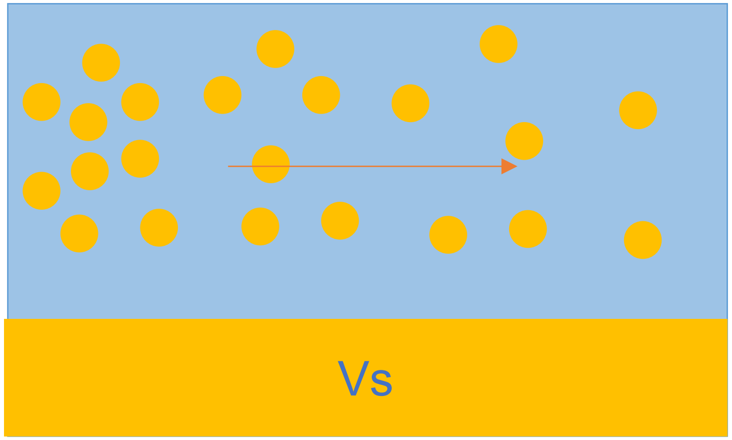
Lee et al. Nature Nanotechnology 2014

1000 larger than for semipermeable membranes!

4) Concentration gradients

Osmotic pressure

Non-selective membranes: interactions with surfaces



Anderson, Ann. Rev of Fluid Mech., (1989)

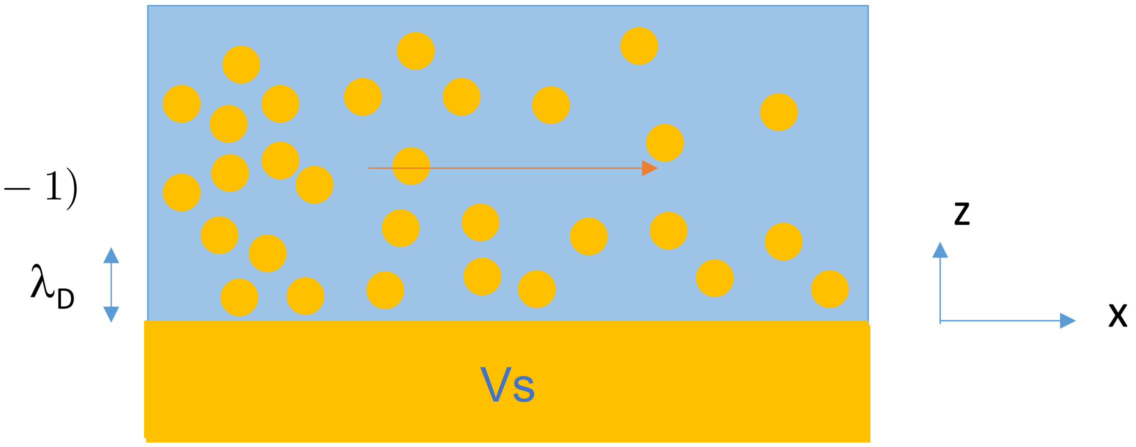
4) Concentration gradients

Osmotic pressure

Non-selective membranes: interactions with surfaces

$$c(z) = c_\infty e^{(-\frac{e\psi(z)}{kT})}$$

$$p(z) = 2c_\infty kT(cosh(\frac{e\psi}{kT}) - 1)$$



$$c_\infty \searrow \quad with\ x$$

$$p \searrow \quad with\ x \longrightarrow$$ Flux

4) Concentration gradients

Diffusio-osmosis

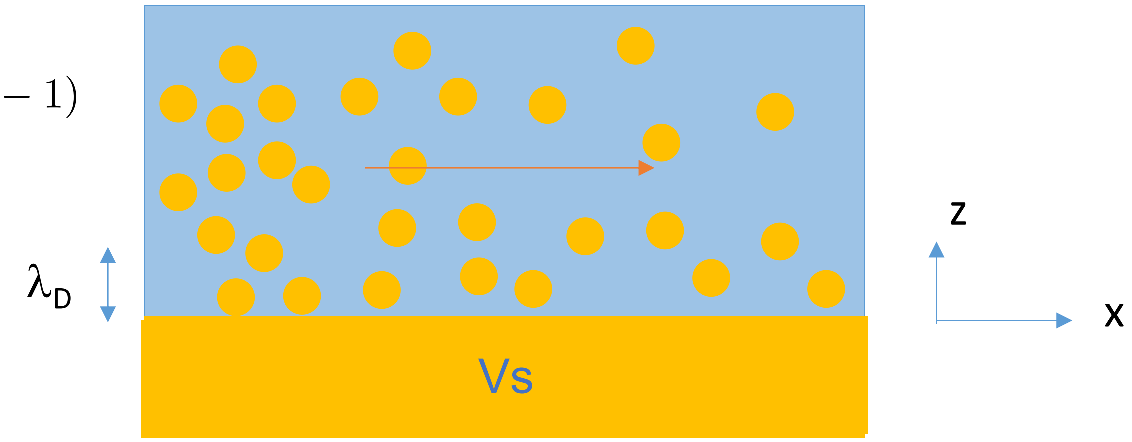
$$\frac{\partial p(x,z)}{\partial x} = \eta \frac{\partial^2 v_x}{\partial z^2}$$

$$c(z) = c_\infty e^{(-\frac{e\psi(z)}{kT})}$$

$$p(z) = 2c_\infty kT(cosh(\frac{e\psi}{kT}) - 1)$$

$$\gamma = tanh(\frac{e\psi_0}{4kT})$$

$$\ell_B = \frac{e^2}{4\pi\epsilon kT}$$

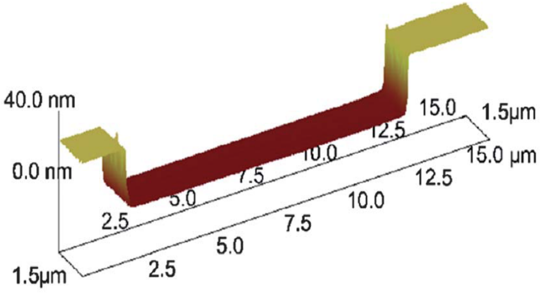


$$\boxed{v_\infty = \frac{kT}{2\pi\eta\ell_B} ln(1-\gamma^2)\frac{dlnc_\infty}{dx}}$$

Plug flow outside the Debye layer

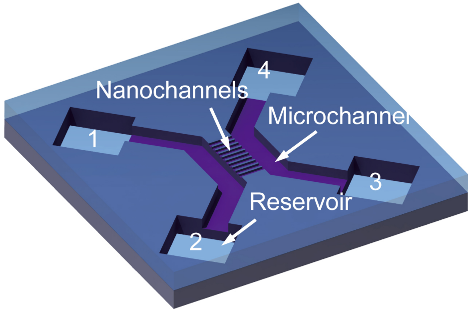
4) Concentration gradients

Diffusioosmosis

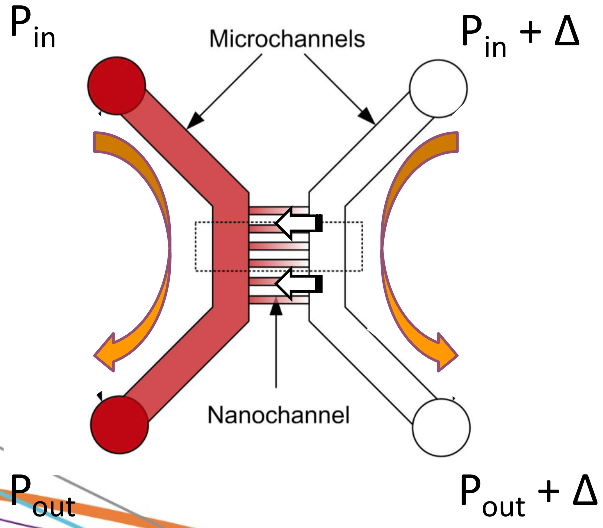


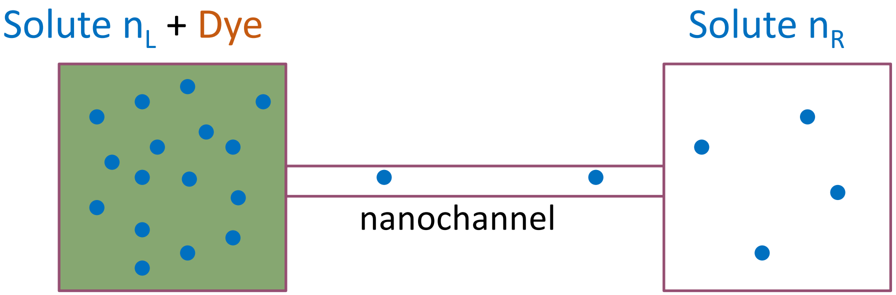
- Lithography
- R. I. E.
- Anodic bonding

h=160nm; w=5µm; L=150µm



Prepare concentration step



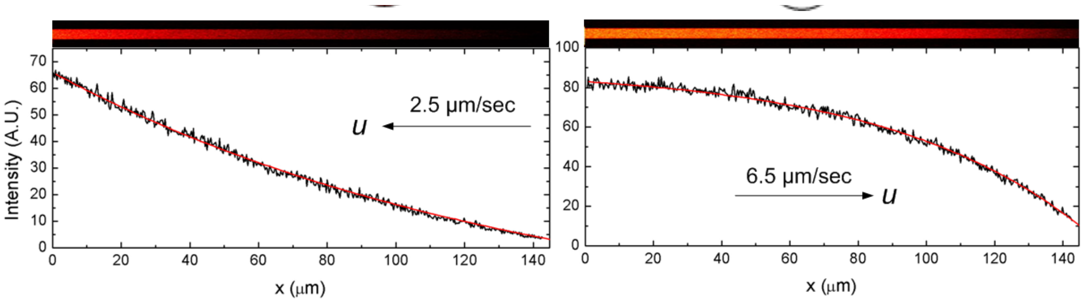


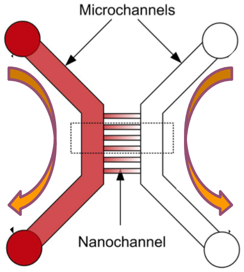
C. Lee, R. Fulcrand, P. Joseph, C. Cottin-Bizonne, C. Ybert

4) Concentration gradients

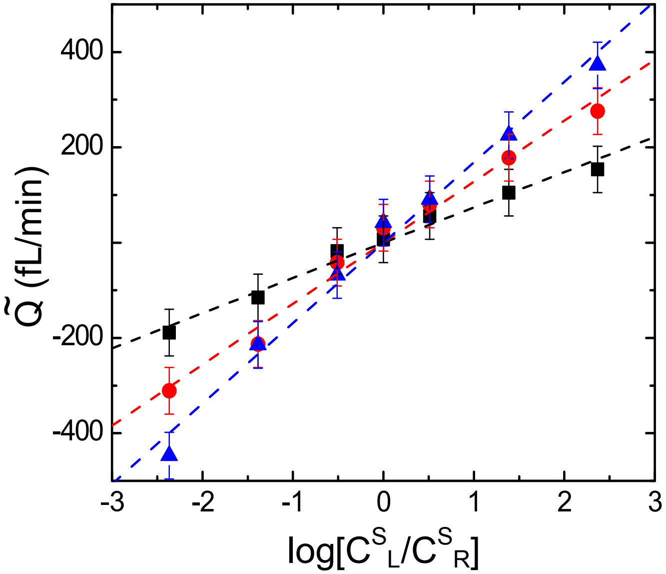
Non-selective $h>\lambda_D$

Diffusioosmosis





LiI, NaI, KI



Plug flow $Q \sim vh$

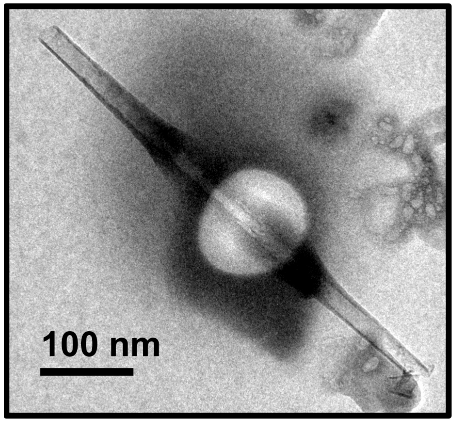
Lee et al., PRL 2014

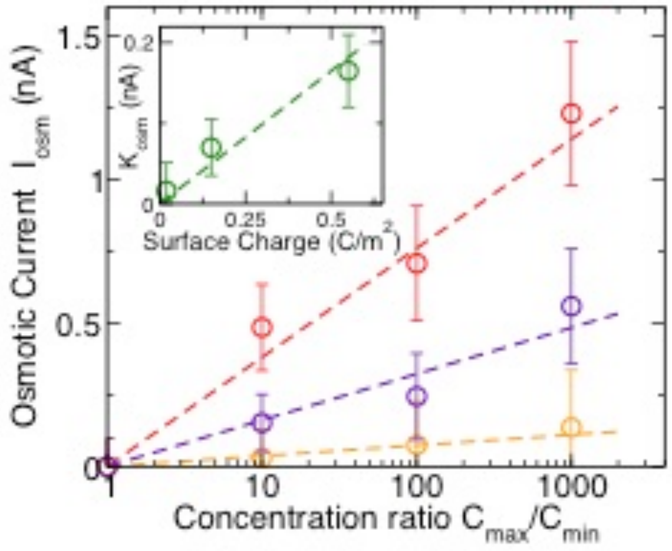
4) Concentration gradients

Diffusioosmotic current

$$I = \int_h e(c_+(z) - c_-(z))v(z)dz$$

$$I_{osm} = 2\pi R\sigma \frac{kT}{2\pi\ell_B}\left(1 - \frac{\ell_{GC}}{\lambda_D} sinh^{-1}\frac{\lambda_D}{\ell_{GC}}\right)\frac{\partial c}{c\partial x}$$





Siria et al., Nature 2013

Expected power density with BN nanotubes: 1kW/m^2

4) Concentration gradients

Comparison diffusioosmosis -electroosmosis

Streaming current

$$I_{streaming} = -\pi R^2 \frac{\Delta P}{L} \epsilon \frac{\zeta}{\eta}$$

Osmotic current

$$I_{osm} = 2\pi R \sigma \frac{kT}{2\pi \ell_B} \left(1 - \frac{\ell_{GC}}{\lambda_D} sinh^{-1} \frac{\lambda_D}{\ell_{GC}}\right) \frac{\partial c}{c \partial x}$$

$$\frac{I_{osm}}{I_{str}} \simeq \frac{kT\Delta c}{\Delta P} \frac{\lambda_D}{R}$$

kTΔc=50 bars !! >> ΔP

4) Concentration gradients

Comparison diffusioosmosis -electroosmosis

Streaming current

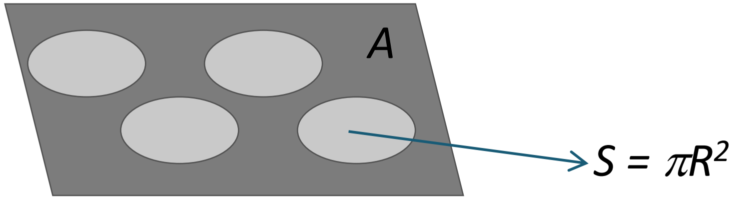
$$I_{str} \sim S \sim R^2$$

Osmotic current

$$I_{osm} \sim R$$

Channel density

$$N \sim A/S$$



$$I_{str,tot} \sim A/S \times S \sim A$$

$$I_{osm,tot} \sim A/S \times R \sim A/R$$

Interesting for small devices!

4) Concentration gradients

QUIZZ

1) Do we need selective membrane to see effects of salinity difference?

2) What is the most efficient: diffusioosmosis or electroosmosis?

3) Do you think that the two can be coupled?

Content

1) Introduction

2) Mass transport

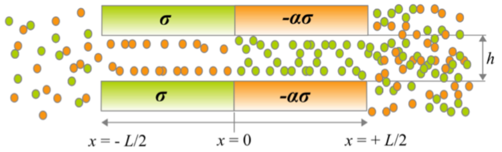
3) Electrokinetics

4) Concentration gradients

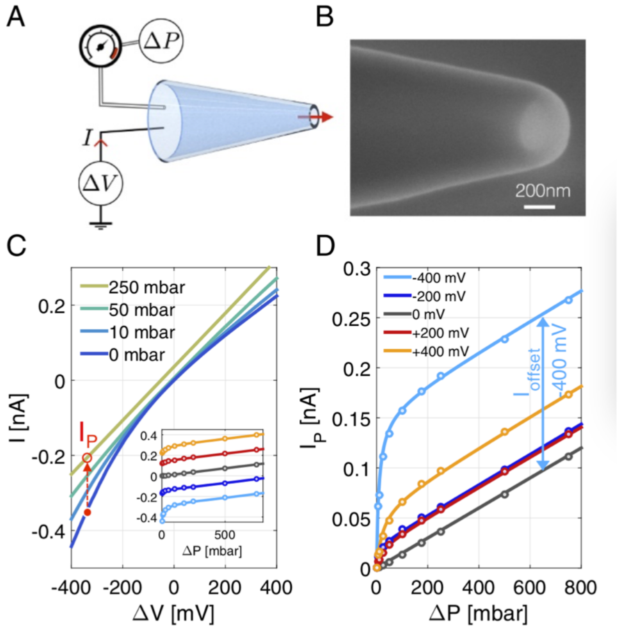
5) Recent issues

4) Recent issues

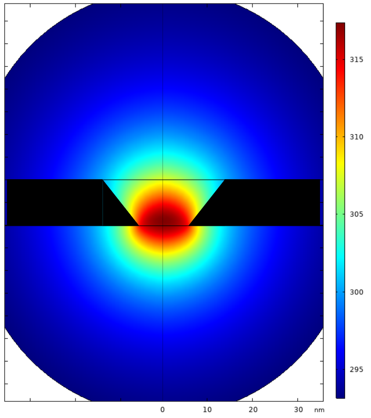
Nanofluidic diodes



Nanofluidic osmotic diode
Picallo et al., PRL 2013



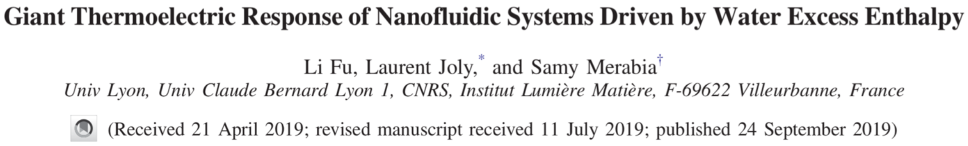
Jubin et al., PNAS 2016



Non-symmetric Joule heating

4) Recent issues

Thermoosmosis



Giant Thermoelectric Response of Nanofluidic Systems Driven by Water Excess Enthalpy

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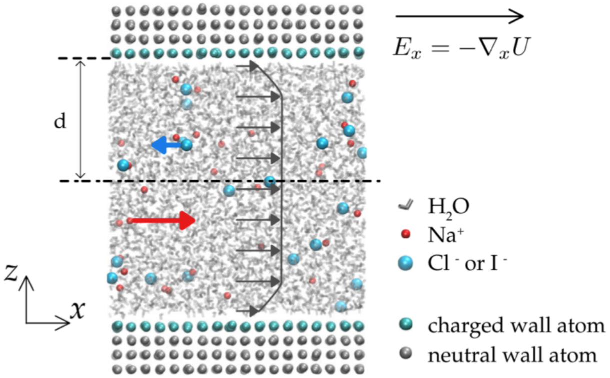
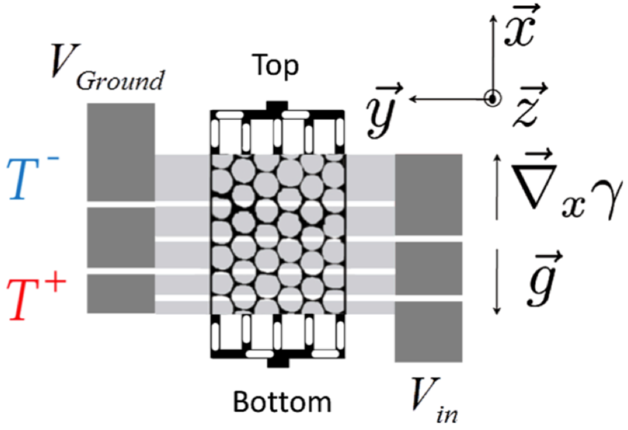
(Received 21 April 2019; revised manuscript received 11 July 2019; published 24 September 2019)





Miralles et al., PRL 2014

THANK YOU

NECTAR Project

QUIZZ: who’s who?
Laurent Joly
Cécile Cottin-Bizonne
Christophe Ybert
Oriane Bonhomme