SETE 2019

Microfluidics: introduction

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Microfluidics in nature

Capillaries networks

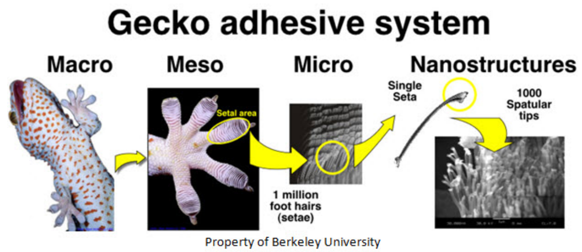
Guell Park, Barcelone

Scaling laws

Quantity	Scaling
Intermolecular forces (Van der Waals)	l^{-7}
Time	l^{0}
Capillary force	l^{1}
Flow velocity	l^{1}
Gravity force	l^{3}

The predominance of forces is different at smaller scales
➡ requires developping fully new plumbing.

Adhesive force : van der Waals attraction

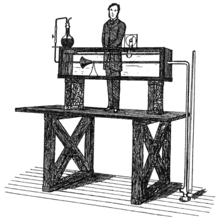


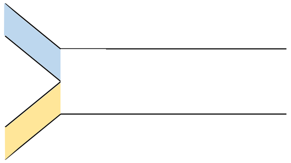
$$F_{vdW} \propto \frac{1}{r^7}$$

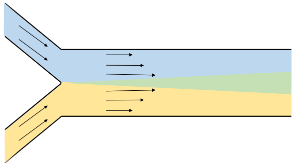
Une sétule peut supporter au maximum une force de 200 μN tout en restant collée à une surface verticale. Ainsi théoriquement, si toutes les sétules avaient une force d’adhésion de 200 μN, la force d’adhésion totale (6,5 millions de sétules) du gecko serait de 1300 N. Le gecko serait donc capable de supporter une masse d’environ 130 kg sur son dos tout en restant collé au plafond.

Flows at the microscale vs. macroscale

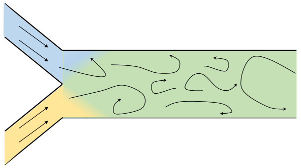
Expérience de Reynolds





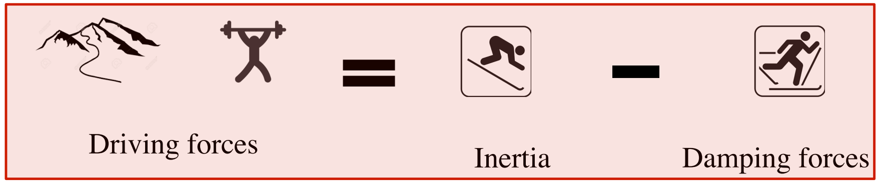


Ecoulement laminaire

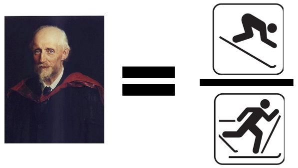


Ecoulement turbulent

Momentum conservation



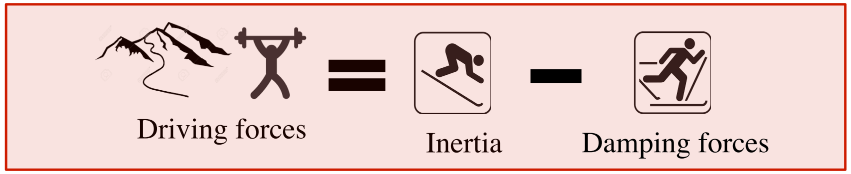
Driving forces = Inertia − Damping forces



$= \mathcal{R}e = \frac{V_{moy}R}{\nu}$

O. Reynolds

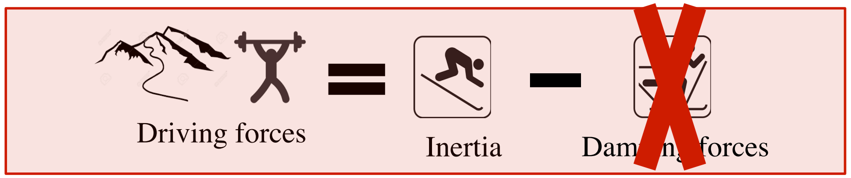
Momentum conservation



Driving forces = Inertia − Damping forces

$\gg 1$

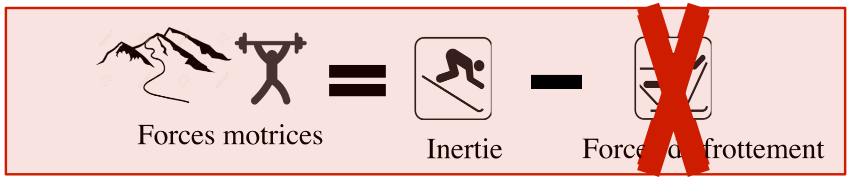
Momentum conservation



Driving forces = Inertia − ~~Damping forces~~

$\propto \frac{VR}{\nu} >> 1$

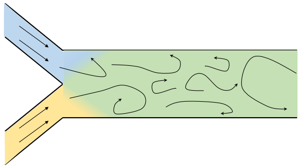
Momentum conservation


Forces motrices

Inertie

Force d frottement

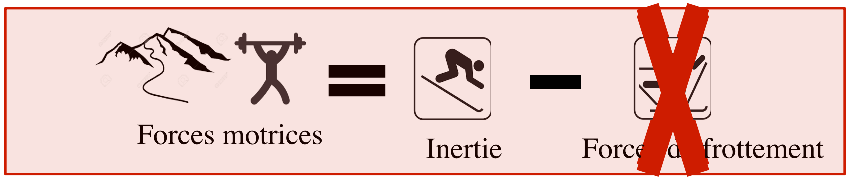

$\propto \frac{VR}{\nu} >> 1$



Ecoulement turbulent

Ecoulements grandes échelles : Grand Y

Momentum conservation


Forces motrices

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Ecoulement turbulent

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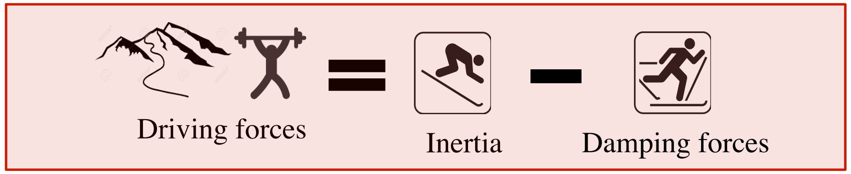
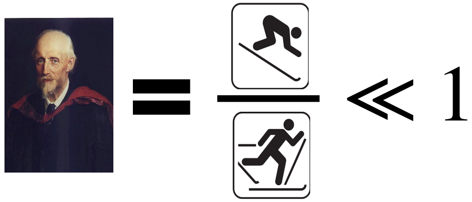
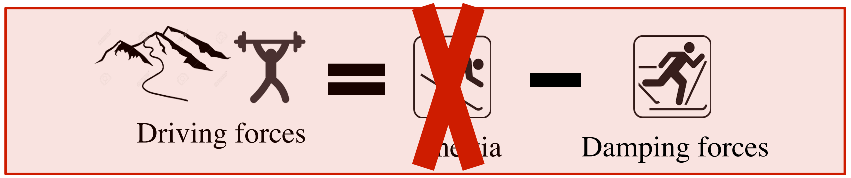

Driving forces

Inertia

Damping forces



Momentum conservation

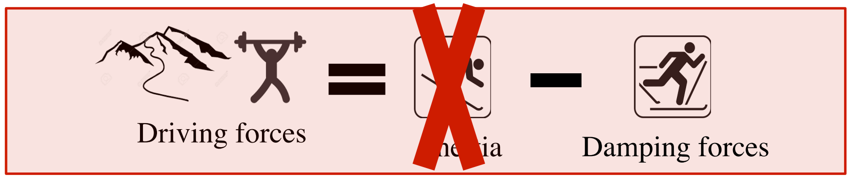

Driving forces

ne ia

Damping forces

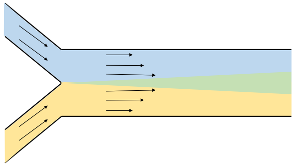

$\propto \frac{VR}{\nu} \ll 1$

Momentum conservation



Driving forces = ~~ne ia~~ – Damping forces

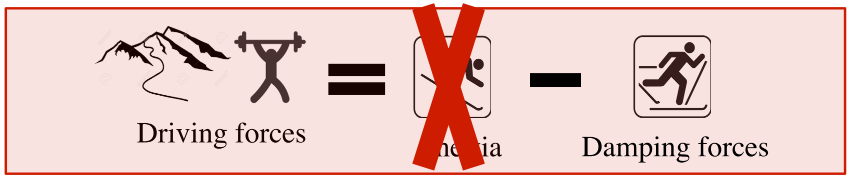
$\propto \frac{VR}{\nu} \ll 1$



Ecoulement laminaire

Ecoulements petites échelles : Micro Y

Momentum conservation



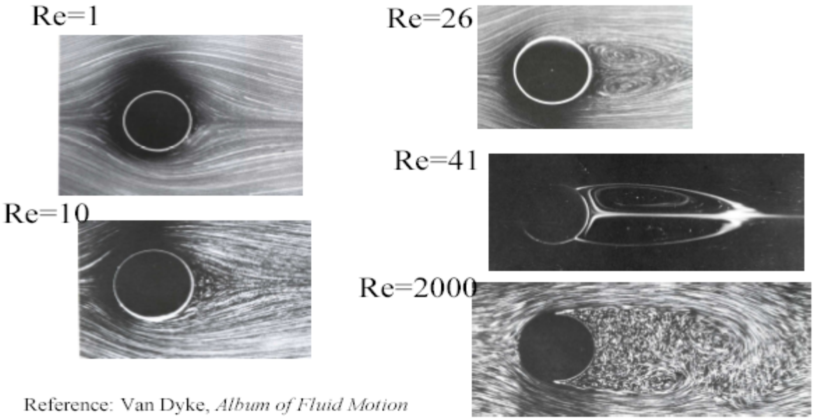
Driving forces = ~~ne ia~~ – Damping forces

$\propto \frac{VR}{\nu} \ll 1$

Ecoulement laminaire

Ecoulements petites échelles : Micro Y

Illustration of the flow patterns for an increasing Reynolds number



Reference: Van Dyke, *Album of Fluid Motion*

Flow at the microscale

Conservation of momentum for a newtonian fluid: Navier-Stokes equation

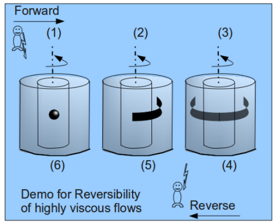
$$\rho\left(\cancel{\frac{\partial \vec{u}}{\partial t}} + \cancel{(\vec{u}\cdot\vec{\nabla})\vec{u}}\right) = -\vec{\nabla}p + \eta\Delta\vec{u} + \cancel{\rho\vec{g}}$$

$$Re = \frac{\left[\rho(\vec{u}\cdot\vec{\nabla})\vec{u}\right]}{[\eta\Delta\vec{u}]} = \frac{uL}{\nu} \ll 1$$

Bulk forces are negligible

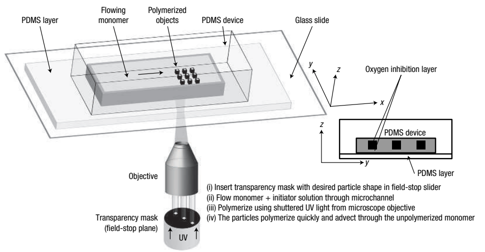
Stokes equation:

$$\boxed{\eta\Delta\vec{u} - \vec{\nabla}p = 0}$$

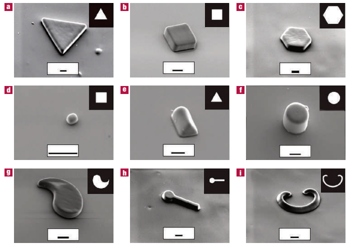


https://home.iitm.ac.in/arunn/laminar-flow-reversibility-why-does-the-blob-rewind.html

Fabrication of particles using laminar flows

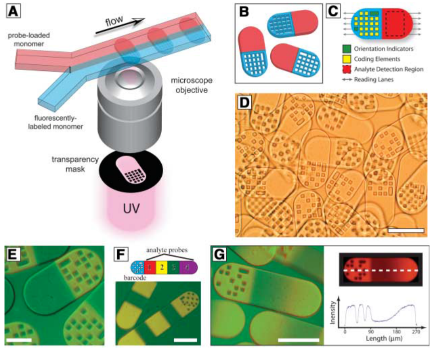


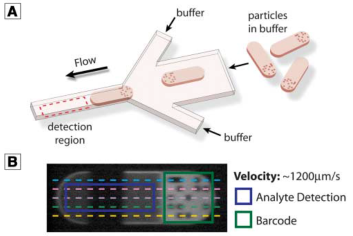
(i) Insert transparency mask with desired particle shape in field-stop slider
(ii) Flow monomer + initiator solution through microchannel
(iii) Polymerize using shuttered UV light from microscope objective
(iv) The particles polymerize quickly and advect through the unpolymerized monomer



Dandukuri et al., Nature Materials, 2006

Fabrication of particles using laminar flows

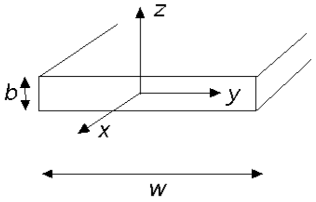




Pregibon et al, Science 315, 2007

The hydrodynamic resistance

$$\eta \Delta \vec{u} - \vec{\nabla} p = 0$$



Hele Shaw cell: $b \ll w \ll L$

$$\Rightarrow \quad \vec{u} = u(z)\vec{x}$$

$$G = -\frac{\partial p}{\partial x}$$

$$\vec{u}(\pm b/2) = 0$$

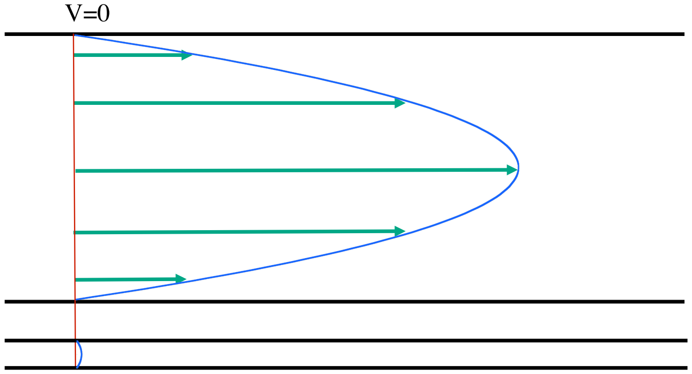
$$\vec{v} = -\frac{G}{2\eta}\left(z^2 - \frac{b^2}{4}\right)$$ Poiseuille flow

$$\Delta p = \boxed{\frac{12\eta L}{wb^3}} Q$$

→ Hydrodynamic resistance

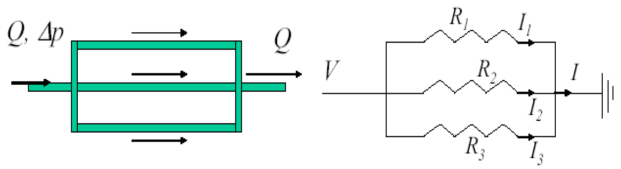
For a cylindrical geometry

$$R_{\text{hydro}} \propto \frac{1}{d^4}$$



Mean velocity 100 times smaller !
Cross section 100 smaller
=> Flow rate 10 000 times smaller

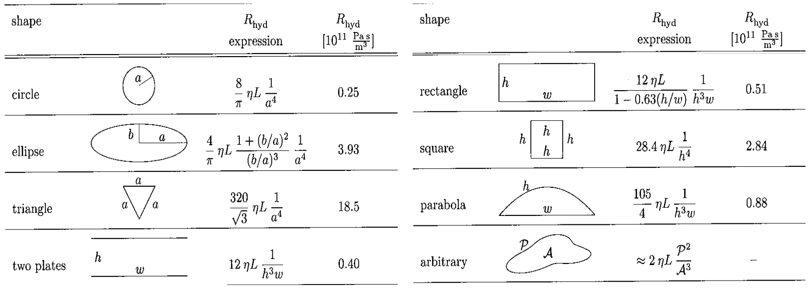
The hydrodynamic resistance



Microchannel network	Electrical equivalent
Pressure	Potential
Flow rate	Electrical current
Hydrodynamic resistance	Electrical resistance

Kirchoff's laws

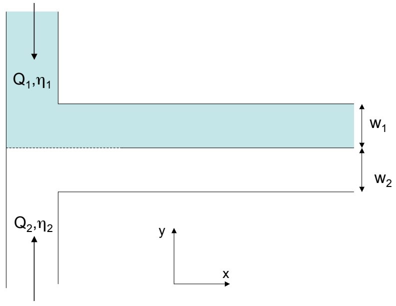
The hydrodynamic resistance



shape	R_{hyd} expression	R_{hyd} $[10^{11}\ \frac{\text{Pa s}}{\text{m}^3}]$
circle	$\frac{8}{\pi}\,\eta L\,\frac{1}{a^4}$	0.25
ellipse	$\frac{4}{\pi}\,\eta L\,\frac{1+(b/a)^2}{(b/a)^3}\,\frac{1}{a^4}$	3.93
triangle	$\frac{320}{\sqrt{3}}\,\eta L\,\frac{1}{a^4}$	18.5
two plates	$12\,\eta L\,\frac{1}{h^3 w}$	0.40

shape	R_{hyd} expression	R_{hyd} $[10^{11}\ \frac{\text{Pa s}}{\text{m}^3}]$
rectangle	$\frac{12\,\eta L}{1-0.63(h/w)}\,\frac{1}{h^3 w}$	0.51
square	$28.4\,\eta L\,\frac{1}{h^4}$	2.84
parabola	$\frac{105}{4}\,\eta L\,\frac{1}{h^3 w}$	0.88
arbitrary	$\approx 2\,\eta L\,\frac{\mathcal{P}^2}{\mathcal{A}^3}$	–

Rheology of newtonian fluids



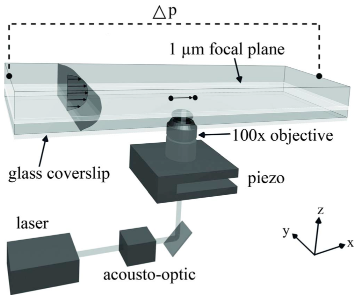
$$\frac{Q\eta}{w} = \frac{b^3 G}{12} \quad \text{est conservé donc} \quad \frac{Q_1\eta_1}{w_1} = \frac{Q_2\eta_2}{w_2}$$

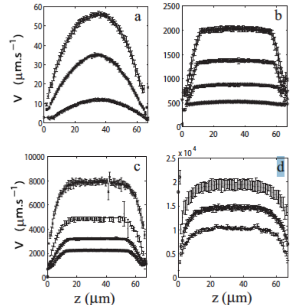
$$\boxed{\frac{w_1}{w_2} = \frac{Q_1}{Q_2}\frac{\eta_1}{\eta_2}}$$

Groisman & Quake, Physical Review Letters, 2004.

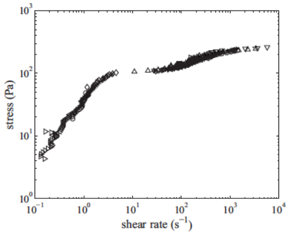
Guillot et al., Langmuir, 2006.

Rheology of complexfluids (wormlike micelles)





0.3M CTAB solution in a
0.405M sodium nitrate
NaNO3 brine



$$\nabla p = \operatorname{div}(\sigma)$$

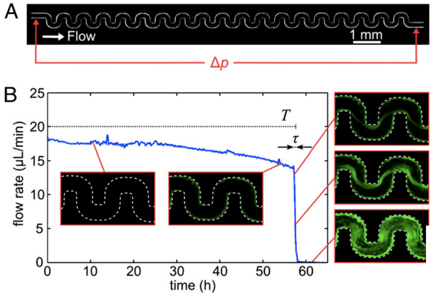
$$\frac{\Delta p}{L} = \partial_y \sigma_{xy} + \partial_z \sigma_{xz}$$

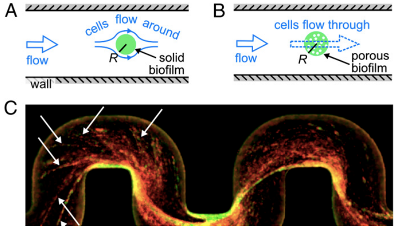
$$\partial_y \sigma_{xy} \ll \partial_z \sigma_{xz} \quad \Rightarrow \quad \boxed{\sigma_{xz} \propto \frac{\Delta p}{L} z}$$

Nghe et al., APL 2008

Flows are not always simple laminar flows

pathogen Pseudomonas aeruginosa

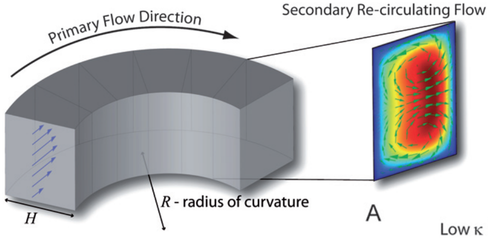


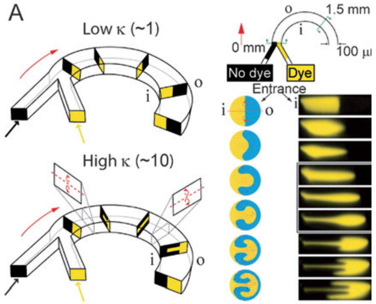


Biofilm streamers that initiate on corners rapidly expand.
Wall-attached biofilm is a necessary precondition for streamer formation

Drescher et al., PNAS 2012

Secondary flows





Dean number :

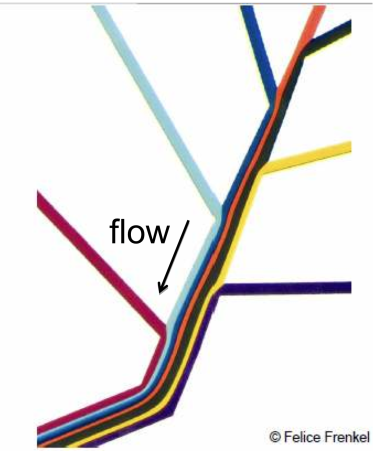
$$\kappa = (H/2R)^{1/2} Re$$

Full Navier-Stokes equation is required

Di Carlo, Lab Chip, 2009

MIXING

At low Reynolds numbers



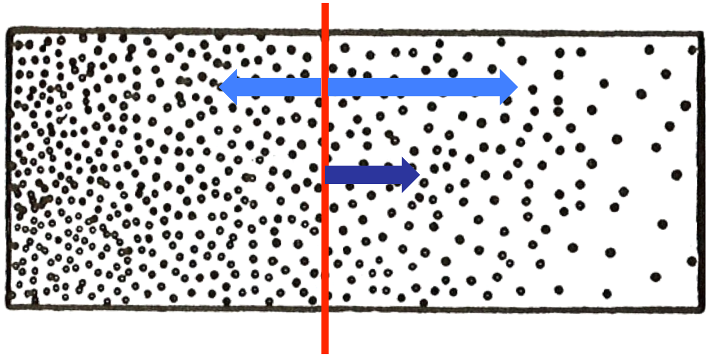
Negligible diffusion

No turbulence

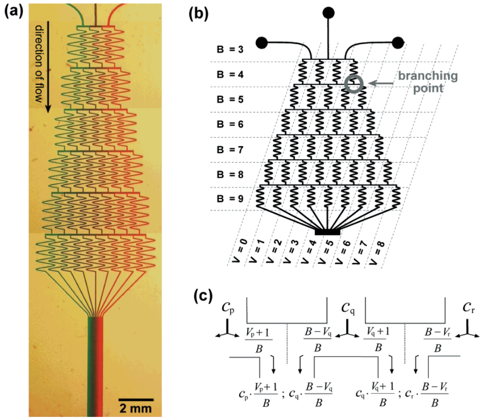
Nearly two dimensional flow

NO MIXING!

Diffusion with « hands »

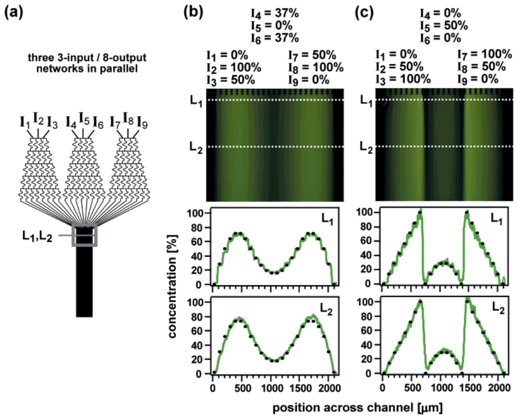


Generating concentration gradients



Dertinger et al., Anal. Chem. 2001, 73, 1240-1246

Generating concentration gradients



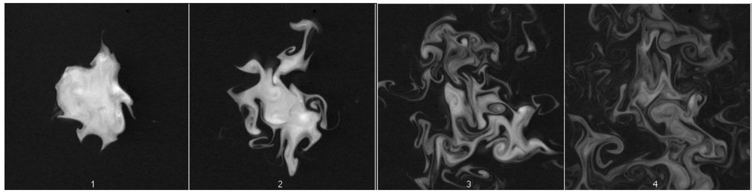
Dertinger et al., Anal. Chem. 2001, 73, 1240-1246

How to mix at low Reynolds numbers?

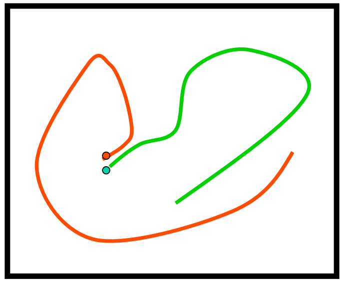
Convection-diffusion equation

- Convection equation for an incompressible liquid :

$$\frac{\partial C}{\partial t} + \vec{u} \cdot \vec{\nabla} C = D \Delta C + q$$

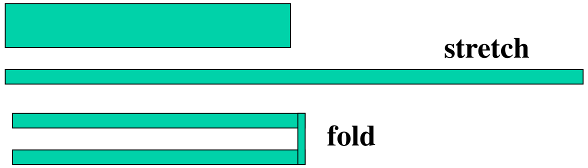


In chaotic regimes, two close particles separate exponentially



Mixing in a non stationary flow : Chaotic regimes

A regime is said chaotic if ,

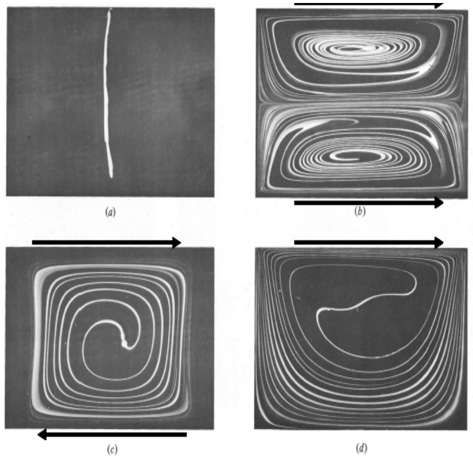


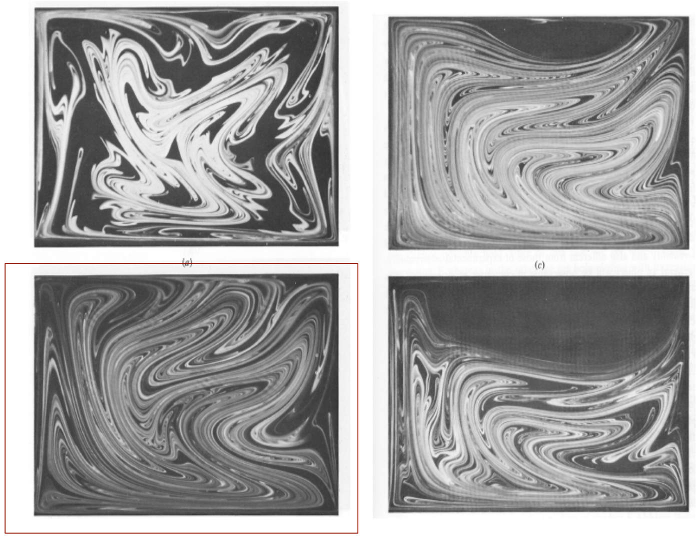
stretch

fold

And so on… (baker transformation)

- Glycérine, Re=1

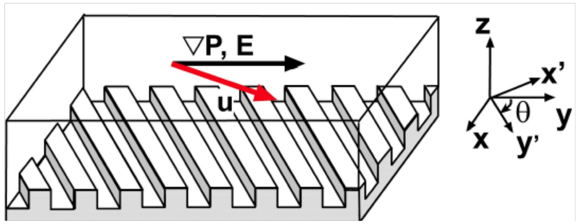




From Ottino's book : « Chaotic Advection »

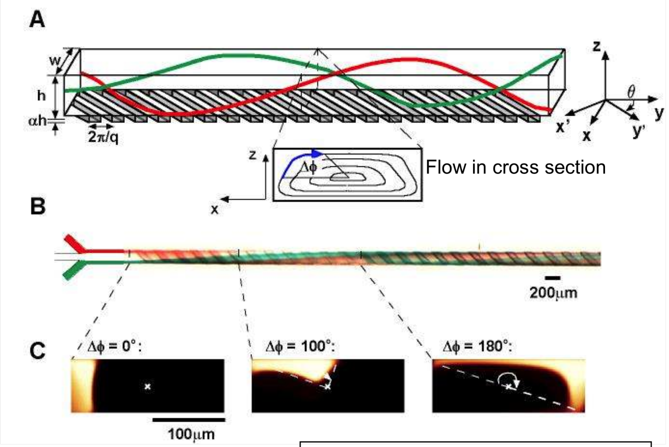
Passive chaotic micromixer

Corrugations produce a three-dimensional flow!



Stroock et al., Sciences 2004

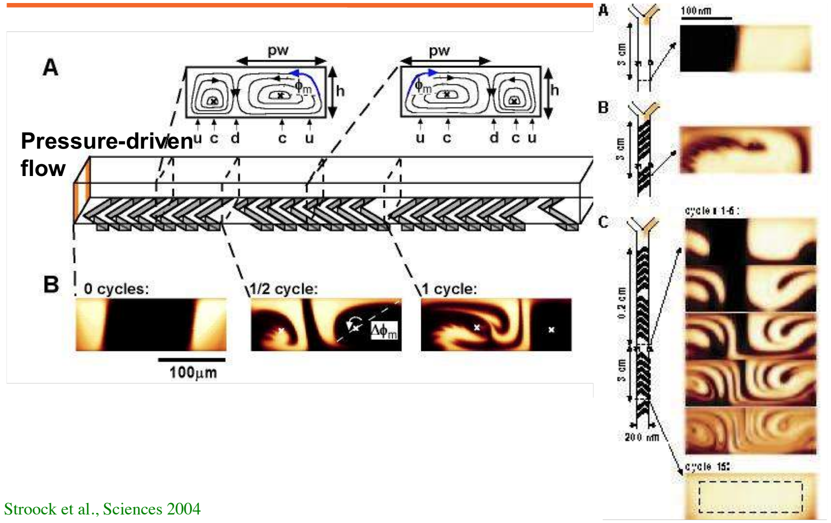
Passive chaotic micromixer



Cross-sectional images (confocal microscopy)

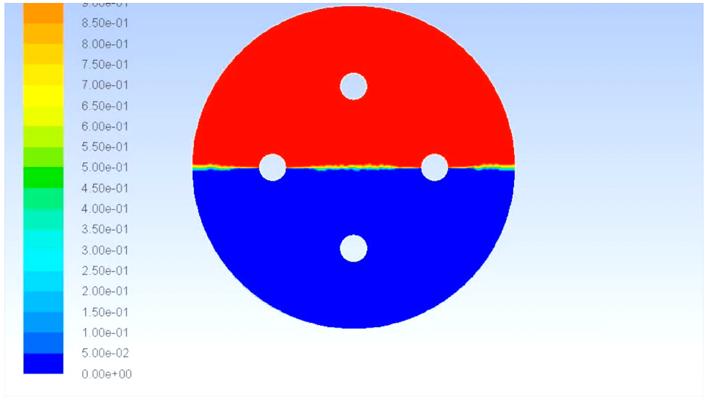
Stroock et al., Sciences 2004

Passive chaotic micromixer



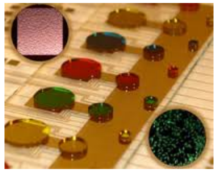
Stroock et al., Sciences 2004

Active chaotic micromixer



Droplet based microfluidics

Droplet-based microfluidics



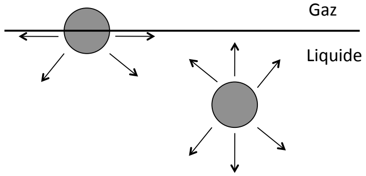
- Applications :
  - droplet = unit system
  - Microreactor
  - Decrease of cross-pollution (appart liquid/liquid extraction)

➔ Necessicity to controle elementary operation:
fabrication, fusion, breakup, transport, storage…

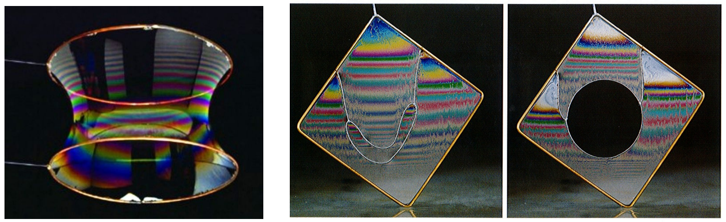
- Models : microporous (stone, lung..)

Definition: surface tension

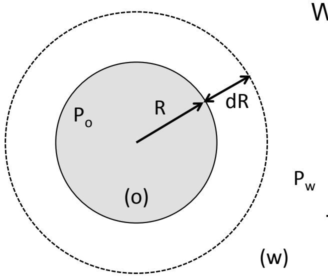
$$\delta W = \gamma \mathrm{d}\mathcal{A}$$



γ Is the surface tension. Its is an energy per unit surface or a force per unit length.



Laplace law demonstration for a sphere



Work to increase the radius by dR:

$$\delta W = -p_o dV_o - p_w dV_w + \gamma_{ow} d\mathcal{A}$$

$$dV_o = 4\pi R^2 dR = -dV_w$$

$$d\mathcal{A} = 8\pi R dR$$

The mechanical equilibrium writes:

$$\delta W = 0$$

$$\Rightarrow \boxed{\Delta p = p_o - p_w = \frac{2\gamma_{ow}}{R}}$$

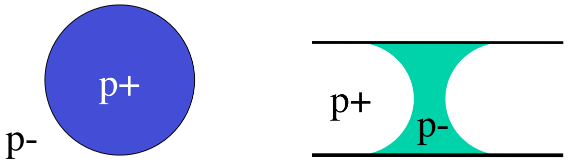
Laplace law : sphere

$$\Delta P = \frac{2\gamma}{R}$$

➡ Capillary bridge of 10 microns

$$\Delta P \sim 2 \times 30.10^{-3}/10^{-5} \sim 6\ \mathrm{kPa}$$

Equivalent to a water column 60 cm height



Gravity vs. Capillary forces

- Bond number

$$\mathrm{Bo} = \frac{\text{gravity force}}{\text{capillary force}} = \frac{\rho l^3 g}{\gamma l} = \frac{\rho l^2 g}{\gamma}$$

- Capillary length

$$l_c = \sqrt{\frac{\gamma}{\rho g}} \sim 2\ \mathrm{mm}$$

---

in situ fabrication

- *Capillary number:*

Continuous phase viscosity

$$Ca = \frac{\mu U}{\gamma}$$

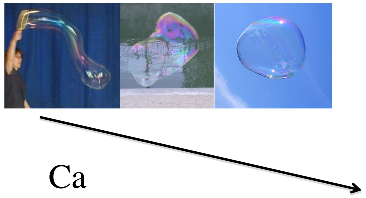
Droplet velocity

Surface tension

$Ca: 10^{-3} - 10^{-1}$

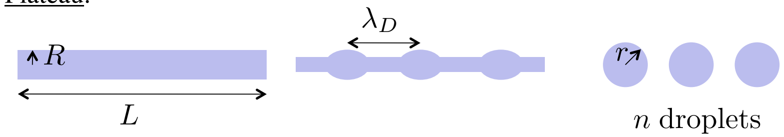
hight => viscosity dominates

small => surface tension dominates



Rayleigh-Plateau instability

Plateau:



Conservation of volume: $\pi R^2 L = \frac{4}{3}\pi r^3 n$

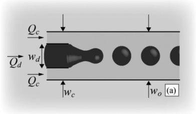
Surface ratio: $\frac{S_n}{S_0} = \frac{n \times 4\pi r^2}{2\pi R L} = \frac{3R}{2r}$

For $r > \frac{3}{2}R$: total droplet surfaces smaller than the one of the liquid thread

---

Droplet in situ fabrication

- Co-axial flow:

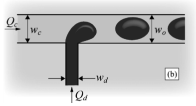


Absolute instability:
⇨ dripping

Convective instability:
⇨ jetting

Guillot et al., PRL 2007

- T-junction:



$w_d/w_c \ll 1$

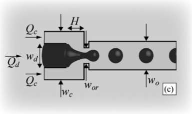
$\Rightarrow L = f(Ca)$

$w_d \sim w_c$

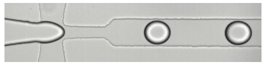
$\Rightarrow L \propto Q_d/Q_c$

Thorsen et al., PRL 2001

- Hydrodynamic focusing:



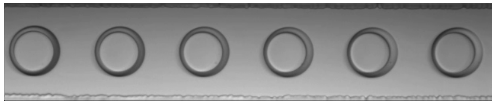
$$V_b = \frac{P_d}{\eta_c Q_c}$$

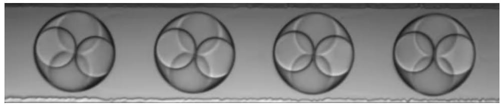


Anna et al., APL 2003
Garstecki et al., APL 2004
Guillot et al., PRE 2008

Fabrication of more complex systems

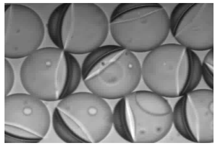
Encapsulation:



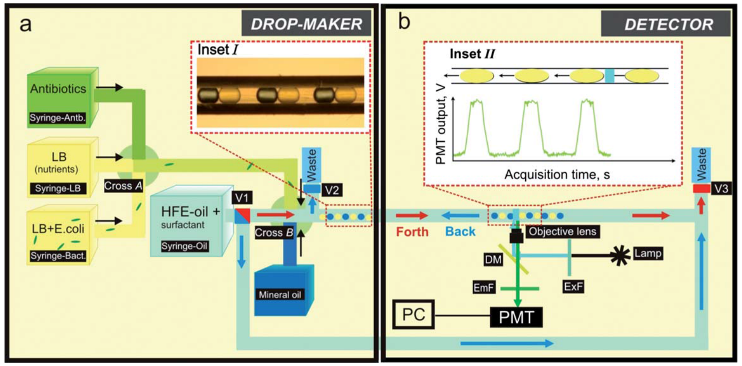


Pannacci et al., PRL 2008

Janus droplet:

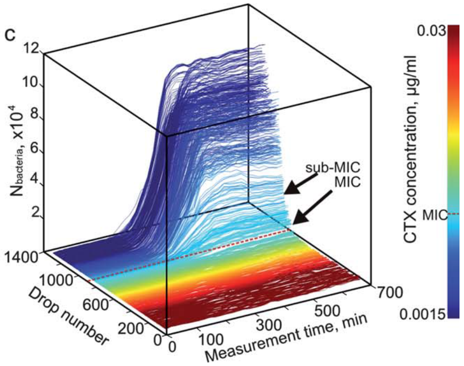


Using droplets for antibiograms



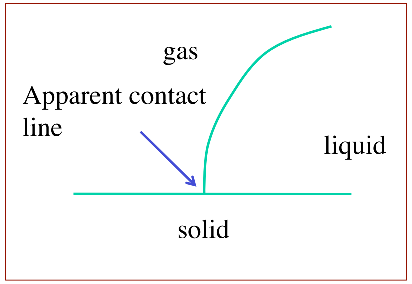
Baraban et al. Lab Chip 2011

Using droplets for antibiograms



Baraban et al. Lab Chip 2011

Wetting film and triple line

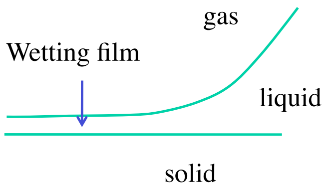


gas

Apparent contact line

liquid

solid



gas

Wetting film

liquid

solid

The spreading coefficient

Gas

Solid

γ_{sg}

Gas

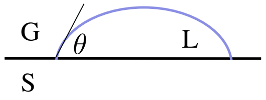
Liquid

Solid

$\gamma_{lg} = \gamma$

γ_{sl}

$$S = E_{dry}^{substrate} - E_{wet}^{substrate}$$

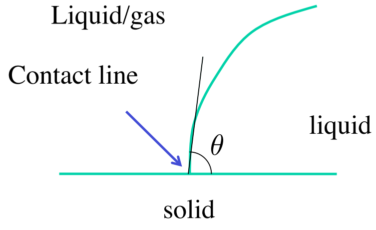


$$\boxed{S = \gamma_{sg} - \gamma_{sl} - \gamma}$$

$S > 0$ Total wetting

$S < 0$ Partial wetting or desorption

Two cases



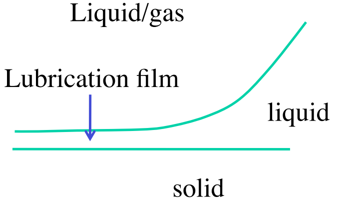
Liquid/gas

Contact line

θ

liquid

solid



Liquid/gas

Lubrication film

liquid

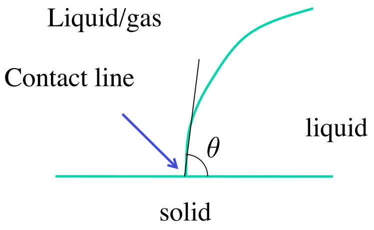
solid

$\theta = 0$

$\theta < \frac{\pi}{2}$ Wetting

$\theta > \frac{\pi}{2}$ Non wetting

Two cases

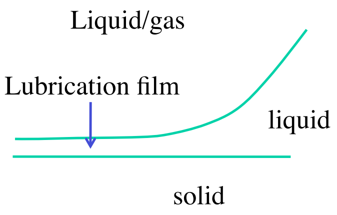

Liquid/gas

Contact line

liquid

θ

solid


Liquid/gas

Lubrication film

liquid

solid

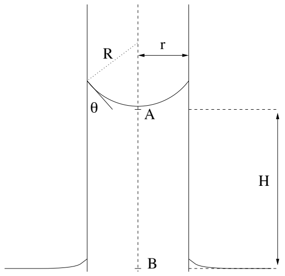
$\theta < \frac{\pi}{2}$ Wetting

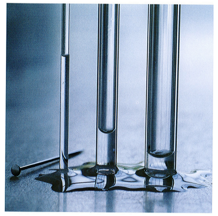
$\theta > \frac{\pi}{2}$ Non wetting

$\theta = 0$


?

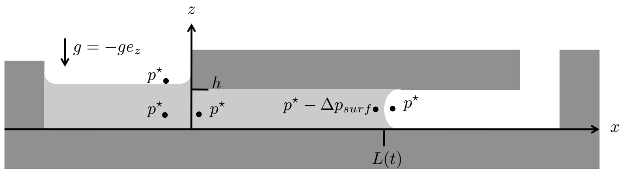
Capillary imbibition: vertical tube – Jurin law





$$\left.\begin{aligned} R &= r/\cos\theta \\ P_A &= P_0 - 2\gamma\cos\theta/r \\ P_A + \rho g H &= P_B \end{aligned}\right\} \quad \boxed{H = \frac{2\gamma\cos\theta}{\rho g r}}$$

Capillary imbibition: horizontal tube – Washburn



Driving force: $\Delta p_{surf} = \frac{2\gamma}{h}\cos\theta$

Viscous force: $\Delta p = (12\mu L/h^3 w)Q$

$$\frac{\mathrm{d}L(t)}{\mathrm{d}t} = V_0 = \frac{Q}{wh} \sim \frac{h^2 \Delta p_{surf}}{12\mu}\frac{1}{L(t)}$$

$$\boxed{L(t) = h\sqrt{\frac{t}{\tau_{adv}}} \quad \text{with} \quad \tau_{adv} \equiv \frac{3\mu h}{\gamma\cos\theta}}$$

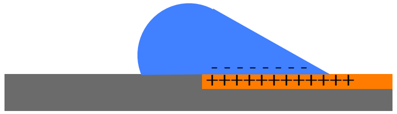
Washburn, Phys. Rev. 1921

Electrowetting : triple line

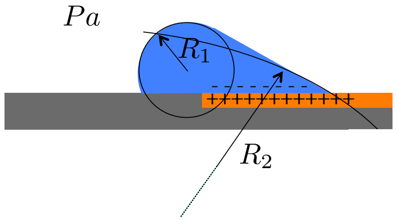
100µm

Electrowetting - EWOD

Electrowetting - EWOD



Electrowetting - EWOD



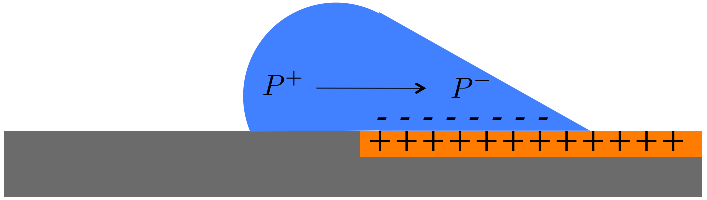
$$\Delta P = \frac{2\gamma}{R}$$

$$P_1 - Pa = \frac{2\gamma}{R_1}$$

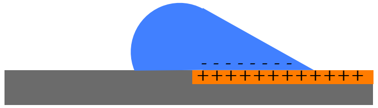
$$P_2 - Pa = \frac{2\gamma}{R_2}$$

$$R_1 \ll R_2 \Rightarrow P_1 \gg P_2$$

Electrowetting - EWOD



Electrowetting - EWOD



More formally, Young-Lippmann equation:

$c = \varepsilon\varepsilon_0 / d$

ε Dielectric constant of indsulator

ε_0 Permittivity of vacuum

d Thickness of insulating layer

Capacitance per unit area between the drop and the electrode

$$\cos\theta = \cos\theta_{\mathrm{Y}} + cU^2/2\gamma = \cos\theta_{\mathrm{Y}} + \eta$$

Voltage-dependent contact angle

Contact angle at zero voltage

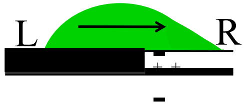
relative strength of electrostatic and surface tension forces

Mugele, Soft Matter, 2009

Mugele & Baret, J. Phys.: Condens. Matter, 2005

Electrowetting

Set up:

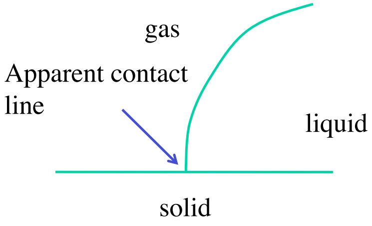


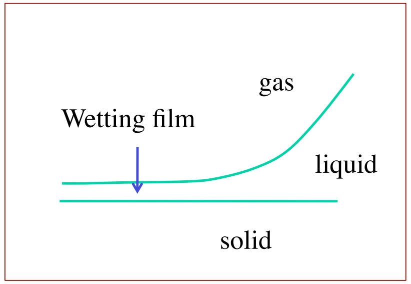
$\theta_R < \theta_L \Rightarrow r_R > r_L$

$$\boxed{P_L - P_R = \frac{\gamma_{LG}(\cos\theta_R - \cos\theta_L)}{R} > 0}$$

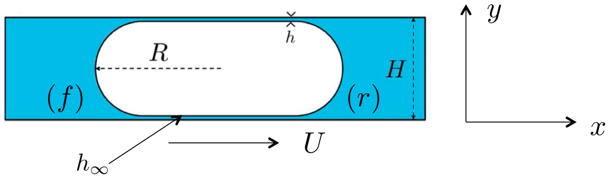
http://microfluidics.ee.duke.edu/

Wetting film and triple line





State of the art: the lubrication problem



Cantat, Physics of fluids, 2013

State of the art

Pressure drop along the droplet in the lubrication film (meniscus + flat film):

$$\begin{cases} \delta p^{sf} = 4.94 Ca^{2/3} \dfrac{4\gamma}{H} \\ \delta p^{sli} = \left(6.82 Ca^{2/3} + 0.94 \dfrac{2l}{H} Ca^{1/3}\right) \dfrac{4\gamma}{H} \\ \delta p^{rol} = 6.22 Ca^{2/3} \dfrac{4\gamma}{H} \end{cases}$$

Pressure drop in the sheared lubrication film

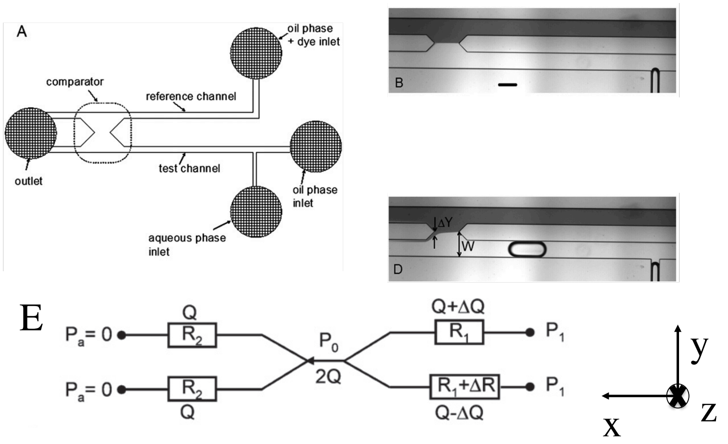
Comparison, pressure drop in the stress-free case vs. Poiseuille flow

$$\delta p^{\mathrm{Pois}} = 8\eta L_t U / r_t^2$$

$$r_t = 1\ \mathrm{mm},\ Ca = 10^{-3} \implies \delta p^{sf} \sim \delta p^{\mathrm{Pois}} \text{ for } L_t = 12\,\mathrm{mm}$$

Cantat, Phys. Fluids, 2013

State of the art



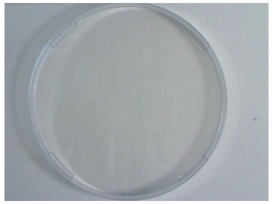
Vanapalli et al., Lab Chip, 2009

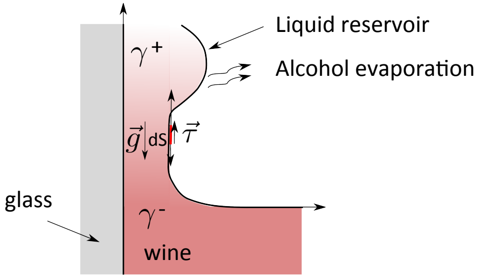
Generating surface flow

Surface tension depends on:

- Solutes concentration
- Temperature
- electric potential

A kitchen experiment:

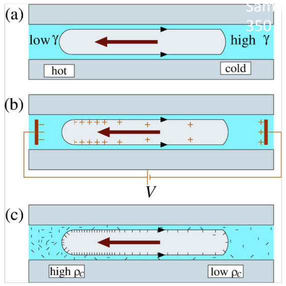




MARANGONI EFFECTS

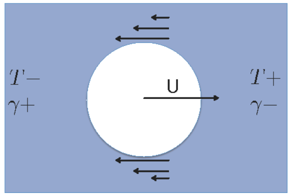
Droplet displacement

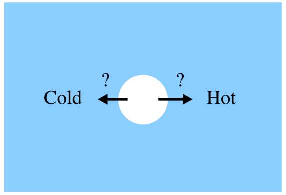
- Wetting film:
  a) Thermocapillarity
  b) Electrocapillarity
  c) Solutocapillarity



Thermocapillarity: state of the art (wetting film)

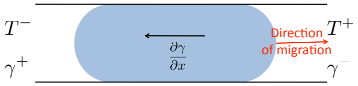
Authors	Geometry	Scale	Direction
Young, Goldstein (1959)	3D	micron	Hot, low γ
Sammarco (1999) Lajeunesse (2003)	1D (tube)	mm	Hot, low γ
Brathukin and Zuev (1984)	2D (Hele-Shaw cell)	mm	Hot, low γ
Selva et al.	2D (Hele Shaw)	micron	**Cold, high γ**





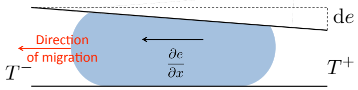
Involved machanisms

Thermocapillary



Towards hot

Thermomechanical



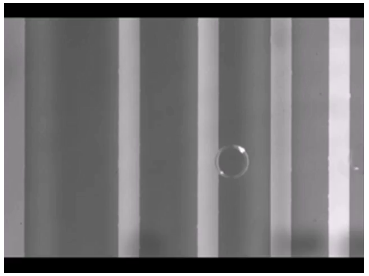
Towards cold

B. Selva et al., Phys. Fluids, 2011

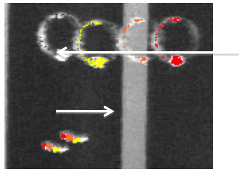
Thermocapillarity vs. thermomechanical

Direction of motion

cold



hot



Thermocapillarity vs. thermomechanical

Thermomechanical actuation: a versatile technology for droplet-based microfluidics

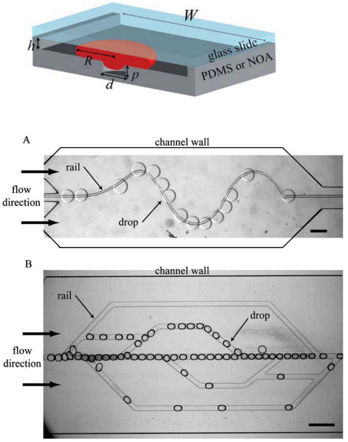
Vincent Miralles [1], Axel Huerre[1], Hannah Williams [1], Bastien Fournié [1], and Marie-Caroline Jullien[1]

[1] MMN, UMR CNRS Gulliver 7083, PSL research University, ESPCI ParisTech, Paris, France

Anchor rails

Displacement

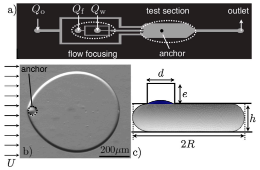
Application : bubble trap



Video related to research article appearing in *Lab on a Chip*

Charles N. Baroud et al. "Rails and anchors: guiding and trapping droplet microreactors in two dimensions"

Read the article at

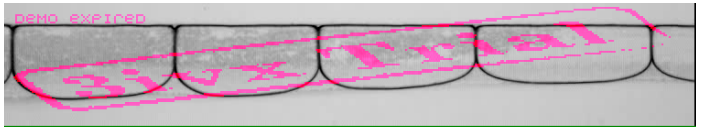


$$F_\gamma \sim \frac{\Delta E_\gamma}{d} \qquad F_d = 24\pi \frac{\mu U R^2}{h}$$

Droplet Fusion

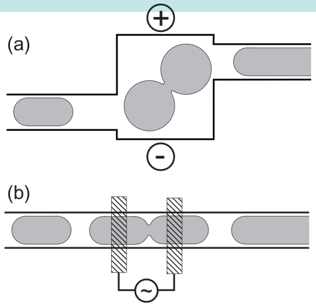
Marie-Caroline Jullien, Microfluidics

Solutocapillary



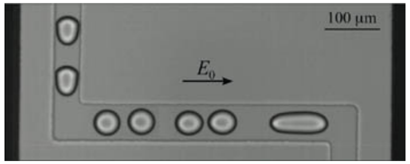
Baroud *et al.*, lab Chip 2007

Electrocoalescence



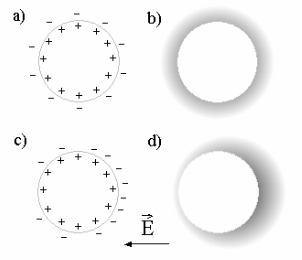
$\sim 1 - 100\ \mathrm{kV.m}^{-1}$

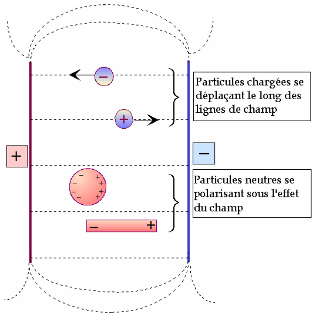
few Volts



Droplet sorting

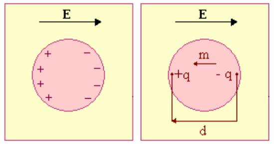
Particle in an electrical field





Particules chargées se déplaçant le long des lignes de champ

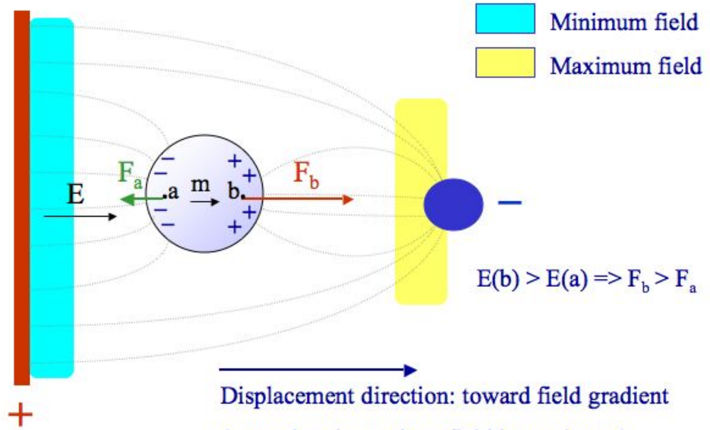
Particules neutres se polarisant sous l'effet du champ



Cell 10 μm for two charges:

Positive dielectrophoresis

Dipolar moment m toward E



🟦 Minimum field

🟨 Maximum field

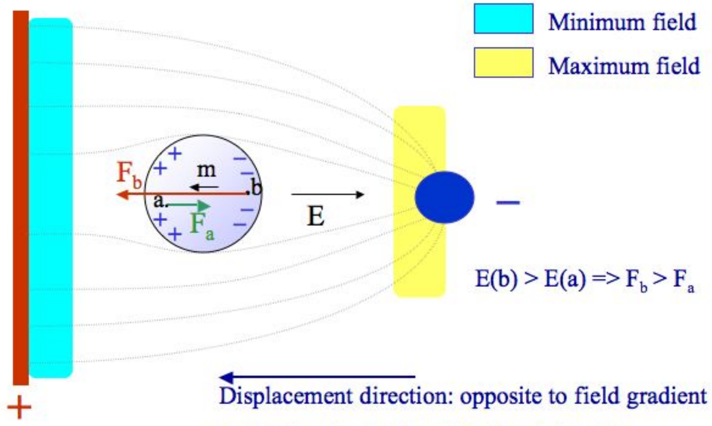
$E(b) > E(a) => F_b > F_a$

Displacement direction: toward field gradient

(toward regions where field is maximum)

Negative dielectrophoresis

Dipolar moment m opposite to E



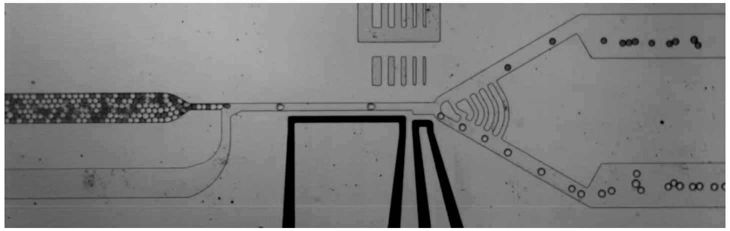
🟦 Minimum field

🟨 Maximum field

$E(b) > E(a) => F_b > F_a$

Displacement direction: opposite to field gradient

(toward regions where field is minimum)

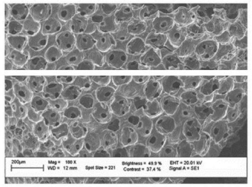


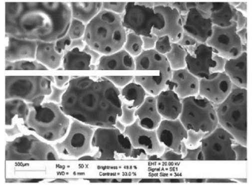
Baret *et al*., Lab Chip 2009

New materials

Droplet-based microfluidics

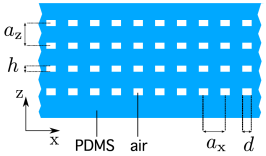
Morphological Comparison of PVA Scaffolds Obtained by Gas Foaming and Microfluidic Foaming Techniques

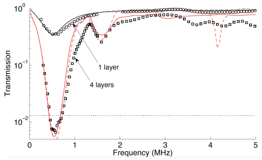




Colsi *et al.*, *Langmuir*, 2013

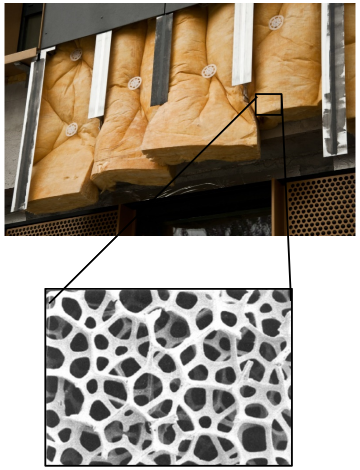
Design and characterization of bubble phononic crystals



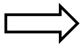


Leroy *et al.*, *APL*, 2009

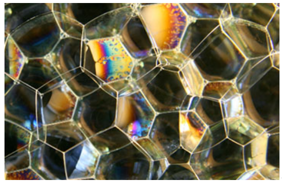
Acoustic insulation



Open cell foam



New paradigm: closed cell foams



Liquid foam